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New Optimized Model Identification in Time Series Model and Its Difficulties

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Abstract

Model identification is an important and complicated step within the autoregressive integrated moving average (ARIMA) methodology framework. This step is especially difficult for integrated series. In this article first investigate Box-Jenkins methodology and its faults in detecting model, and hence have discussed the problem of outliers in time series. By using this optimization method, we will overcome this problem. The method that used in this paper is better than the Box-Jenkins in term of optimality time.

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INTRODUCTION

The increasing importance of forecasts of macroeconomic variables and the lack of structural models for optimizing prediction, time series models have been created. Moreover, identifying models in the shortest possible time and time saving detect of suitable model caused to apply optimizing factor in time series method.

A time series is a sequence of values ordered by a time parameter. The basic goal of time series analysis is to induce from a sample of data points to the process that may have generated the sample. The terms process and time series are equivalent to the concepts of population and sample in classical statistics. A process under study consist of deterministic and stochastic components. Deterministic components are trends and stochastic process is a collection of random variables ordered in time. In the majority of cases , time ordered variables cannot be assume independent , which results in the problem of correlated data , so dependency is expressed by means of the autocorrelation and partial autocorrelation function . Kendall and Buckland (1971) define autocorrelation as correlation between members of series of observation ordered in time, this implies correlation of a series with itself at different lags. The lag k autocorrelation is calculated as:

$$r_k = \frac{\sum_{t=1}^{T-k} (y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum (y_t - \bar{y})^2}$$

t is length and \bar{y} is the means of the series.

In addition of ACF, another function called the partial correlation function (PACF) is employed to describe the memory of a series process. Partial autocorrelation is correlation between y_t and y_{t-k} after removing the effects of intermediate y 's. in general the autocorrelation and partial autocorrelation function are used to define time series

model with dependency structure.

ARIMA modeling

Each times series can be describe three types of mathematical models: autoregressive (AR), moving average (MA) and integrated. In AR process, the value of current observation depend on the previous observation:

$$y_t = \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + u_t$$

(u_t is white noise)

A moving average process is describe by:

$$y_t = u_t - \theta_1 u_{t-1} - \dots - \theta_q u_{t-q}$$

A process containing both autoregressive and moving average components is called mixed. An integrated process is represented by an equation:

$$y_t = y_{t-1} + a_t$$

a_t can be any ARMA process.

Integrated process are nonstationary. Stationary requires that all moment of the series are constant over time. Nonstationary time series must be transformed to stabilize them. The transformation method is dependent on the cause of the nonstationary. Series with a stochastic trends have to be differenced. For those with deterministic trend, polynomial detrending is the correct transformation to achieve stationary. Ayat and Burrige (2000), Elder and Kennedy (2001) describe testing strategies allowing to distinguish between types of nonstationary. Process that are stable after their first differences ($\Delta y_t = y_t - y_{t-1}$) are called integrated of order 1. In general if a time series has to be differenced d time to make it stationary, that series is called integrated of order d. As a consequence an ARIMA (p,d,q) process is identified as an ARMA(P,q) model with P=p+d. this implies that different ARIMA processes such as (1,1,1) and (2,0,0) represent the same ARMA

Table 1: Theoretical ACF and PACF patterns.

Model	ACF	PACF
(0,0,0)	0	0
(p,0,0)	Decay exponentially or with damped sine wave or both	Significant spikes through lag q , 0 after q
(0,0,q)	Significant spikes through lag q , 0 after q	Decline exponentially
(p,0,q)	Decline exponentially	Decline exponentially
(0,d,0)	Does not decay	Does not decay

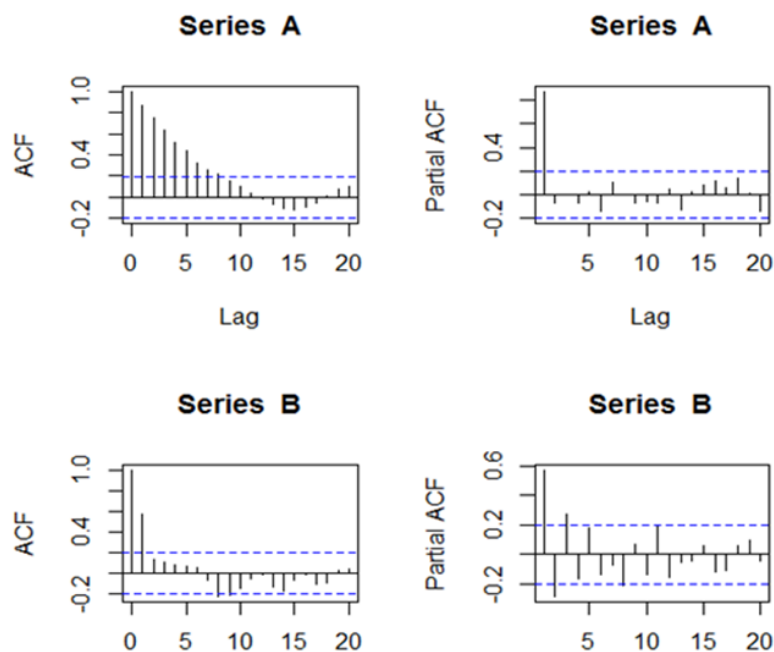


Fig.1. Theoretical ACF and PACF: (A) AR(1) with $\phi=0.99$, (B) MA(1) with $\theta=-0.99$

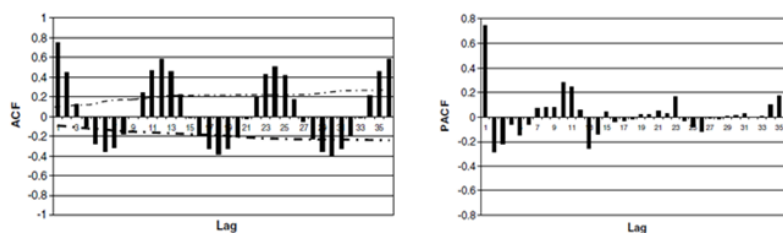


Fig.2. ACF and PACF for Karun river inflow at shaloo bridge station due to Mohammadi et al.

(2,1) model.

Each ARIMA model can be define through its ACF and PACF pattern.

According to the ACF and PACF (fig. 2.), Mohammadi et al. (2006) consider ARMA (2,2) for Monthly river flow data in shaloo bridge station from 1933 to 2001.

MODEL IDENTIFICATION

There exist a number of method for fitting suitable models to a given time series. One of the most widespread techniques is the Box-Jenkins methodology which based on a three steps iterative cycle of model identification, model estimation, and diagnostic checks in model accuracy. At the identification stage one chooses types and order of model examining the behavior of the sample autocorrelation and the sample partial autocorrelation functions and comparing their shapes and value with the theoretical ARIMA patterns.

Fig. 3. illustrates that estimates of ACF and PACF, receive from finite sample, can be rather

ambiguous, even under ideal condition. Furthermore, the quality of empirical ACF and PACF strongly depends on the number of observation in a series and sensitive to outliers. From fig. 3. (E), the Box-Jenkins method is not very useful for identifying mixed ARMA model if p and q are non-zero. The reason for difficulty is that the ACF and PACF of mixed models tail off to identify rather than cut off at a particular lag. Therefore, model identification using the Box-Jenkins approach is a complicated and problematic task requiring many data points and a great deal of expertise from a researcher.

Velicer and Harrop (1983) evaluated the performance of Box-Jenkins model identification technique employing 12 extensively trained subjects and found a disappointing law overall accuracy rate of 28%. The length of a time series (increase from 40 to 100 improved the percent of correct identification from 20 to 36) is the factor affecting the quality of identification and also higher dependency was favorable.

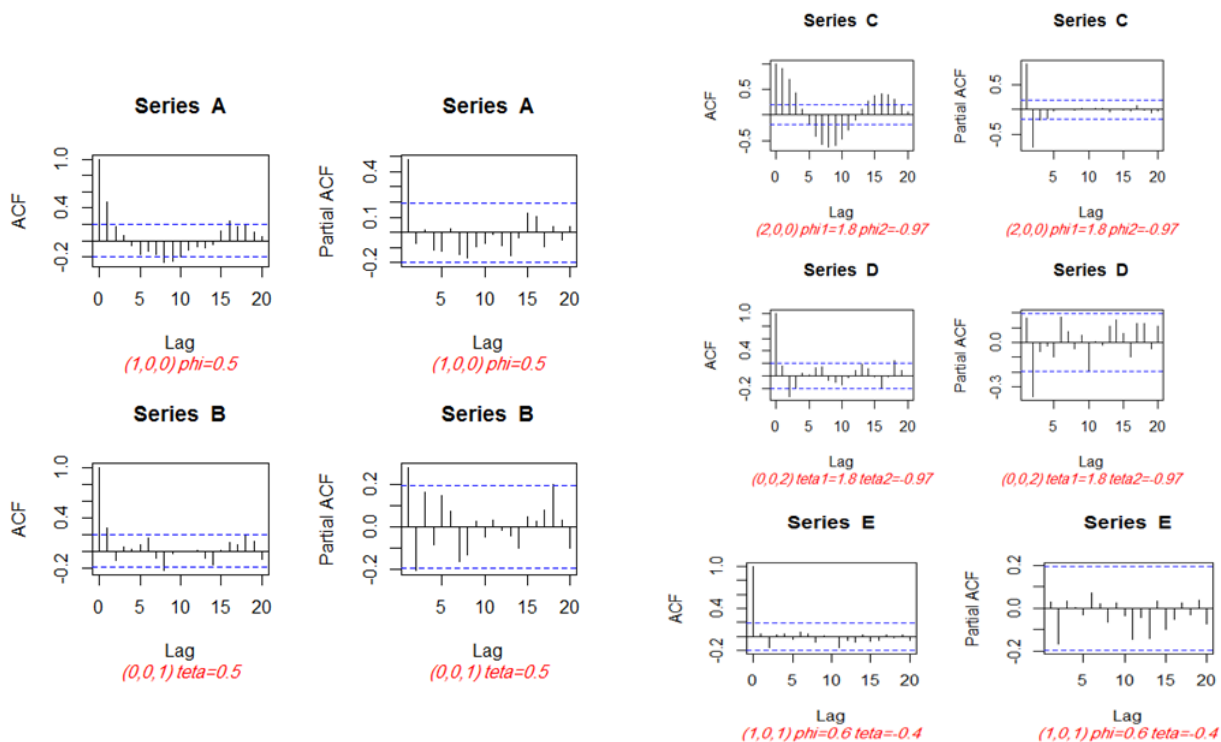


Fig.3. ACF and PACF for simulated models with $n=100$

Identifying integrated models turned out to be the most complicated issue. Extensively trained judges were only to correct identify ARIMA (0,1,0) and (0,1,1) model 4% and 13% of the time respectively. According to Algina and Swaminathan (1977) and Velicer and McDonald (1984), alternative procedure for removal of dependency from the data series have been proposed.

Integrated series comprised a widespread phenomenon among behavioral or psychological series. Glass et al. (1975) reported that out of 95 series taken from a wide range of application in the social science, 44 were non-stationary. Integrated models are typical for process with an infinite memory. For integrated process, the identifying procedure consists of two stage, the first step is to decide whether differencing is necessary or not, and the second stage is to infer the ARMA model by inspecting ACF and PACF of either the original or differenced series.

As previously mentioned, sensitivity to outliers and difficulty in identifying model by using ACF and PACF in mixed models, are two major problem in the Box-Jenkins methodology. In the following, first we investigated ESACF, one the identification method and second analyzed sensitivity to the outliers in brief.

ESACF

The first step, also one of the key steps in building a time series model using ARIMA setting is the order determination step, i.e., identifying p, d and q in the literature, scheme for time series models with identically independent distributed (iid) innovations are as well studied. The Akaike information criterion (AIC) by Akaike (1974) and Bayesian information criteria (BIC) by Schwarz (1978) are two goodness of fit measure of an estimated model to facilitate the model selection, which further extended into AICC, a bias correlated version of AIC by Hurvich and Tsay (1989) and Hannan Quinn information criteria (HQIC) by Hannan and Quinn (1979). Tsay and Tiao (1984) proposed the extended autocorrelation function (EACF) for order determination of ARMA (p, q) model. On the other hand, Dicky and Fuller (1979) studied the unit root behavior and gave the asymptotic distribution of a unit root test statistics. Standard order determination combines those two techniques: taking the unit root test to decide the necessity of making difference (s) (for example, set $y_t = x_t - x_{t-1}$) and then using ACF/PACF/ESACF procedure on differenced series y_t to get AR and MA orders p and q respectively. Other order determination scheme include R and S array approach by Gray-kelly and McIn-

tire , corner method by beguin Gourieroux and Monfort and smallest canonical correlation (SCAN) by Tsay and Tiao (1985) and discussed in Box-Jenkins and Reinsel. Choi (1992) provided comprehensive reviews and descriptions of pattern identification methods and algorithm.

Although autocorrelation function and partial autocorrelation function work perfectly for pure moving average series and autoregressive respectively, however, they do not show strong identification capability for ARMA (p,q) series. Tsay and Tiao proposed the extended autocorrelation function (EACF) technique which can tentatively identify the orders of a stationary or nonstationary ARMA process based on iterated least squares estimates of the autoregressive parameters. Basically, an iterative regression procedure is given to produce consistent estimates of the autoregressive parameters and based on these consistent estimates we utilize the order determination scheme such as ACF for series subtracting the autoregressive term. More specially, the order determination scheme is arranged in the following manner:

1: For each candidate AR order p, we first get the consistent estimates of $\hat{\phi}_1, \dots, \hat{\phi}_p$.

2: Denote $y_t = x_t \hat{\phi}_1 y_{t-1} + \dots + \hat{\phi}_p y_{t-p}$. For each candidate autoregressive order p, given the consistent estimate $\hat{\phi}_1, \dots, \hat{\phi}_p$ we can get y_t from y_1, \dots, y_{t-p} and what remain should be a MA (q) model if we get AR order correctly.

3: By calculating the autocorrelation function of series y_t , we can have the significant test results on every moving average lag q.

4: Marking significant levels on different moving average lags 1 for all candidate AR orders p, we plot the EACF table and choose ARMA or-

ders by identifying the upper right zero triangle. See following table 3. For more details refer to Tsay and Tiao (1984).

Table 3 depicts the theoretical pattern associated with an ARMA (1,2) series. For each pair of AR and MA orders, we test the significance of t-statistics of the autocorrelation function and mark insignificant points as 0, boundary significant points as 1 and significant points as 2. the insignificant points compose the upper right zero triangle and the starting point of triangle's coordinate is (2,3) and thus we identify the model as ARMA(1,2).

Since the most important usage of the EACF table is to determine the tentative ARMA orders, it is of great interest to have an effective automatic algorithm to figure out the most suitable ARMA orders from a given EACF table:

Algorithm: identify autoregressive (AR) and moving average (MA) from a given EACF table.

Denote EACF (i,j) as the significance located at the row and jth column of the EACF table $1 \leq i \leq n, 1 \leq j \leq m$.

Step1: ARMA (i-1, j-1) can be candidate model if:

1) $EACF(i, j) = 0$ And

2) $Real(i, j) \leq Theory(i, j)$ Where

$$Real(i, j) \triangleq \sum_{s=0}^{n-i} \sum_{t=s}^{m-j} (s+i+t+j)^{-0.5} \times EACF(s+i, t+j)$$

$$Theory(i, j) \triangleq \sum_{s=0}^{n-i} \sum_{t=s}^{m-j} (s+i+t+j)^{-0.5} \times 1$$

We want the upper-right triangle leading by EACF(i,j) to beat at least the triangle of 1 to qualify ARMA (i-1, j-1) as a candidate model.

Table 1: The characteristics of unconstrained benchmark functions

AR	MA							
	0	1	2	3	4	5	6	7
0	2	2	2	2	2	2	2	2
1	2	2	0	0	0	0	0	0
2	2	2	2	0	0	0	0	0
3	2	2	2	2	0	0	0	0
4	2	2	2	2	2	0	0	0

2= significant
1= boundary significant
0= insignificant

Table 3: The approximate ESACF table for monthly river flow data in shaloo bridge station from 1933 to 2001.

AR	MA						
	0	1	2	3	4	5	...
0	X	X	X	X	X	X	...
1	X	X	X	X	X	X	...
2	X	X	0	0	0	0	...
3	X	X	X	0	0	0	...
4	X	X	X	X	0	0	...
...

Step 2: Best ARMA models among those qualified candidate can be found. Set $k=i+j$ and enumerate k from 2 to $m+n$.

Now, if there is only one qualified candidate then this is the final model, and if more than one qualified candidates, the one which has lower *Real/theory* is the best model, and if no qualified candidates set $k:=k+1$.

As a results from above, and due to Mahammedi et.al, ESACF has correctly recognition the model (ARMA (2,2)):

OUTLIERS

There are many informal definition of an outlier but only a few, recently published. According to Barnnet and lewis (1994): "an outlier in a set of data is an observation or a patch of observation which appears to be inconsistent with the remainder of that set of data." Regression and time series models have their own types of outliers. Fox (1972) define two types of outliers in the time series context, Type I and type II which are known as additive (AO) and innovational (IO) outliers.

A. Additive outlier (AO)

An additive outlier is an events that affects a series for one time period only, its effect are independent of the ARIMA model. If we assume that an outlier occurs at time = T , and y_t be a time series following an ARIMA model, we have the AO model

$$z_t = y_t + o_A P_t^{(T)}$$

Where $P_t^{(T)} = 1$ when $t=T$, 0 otherwise, and o_A is the magnitude of the isolated, additive outlier.

B. Innovational outlier (IO)

An innovational outlier is an event whose effect is propagated according to the structure of the

ARIMA model of y_t . This model is

$$z_t = y_t + U(B)^{-1} \frac{\theta(B)}{\phi(B)} o_I P_t^{(T)}$$

Where o_I is the magnitude of a single innovational outlier at time = T .

Additive outlier are in practice more common than innovational outliers. For statistical analysis, the AOs are more dangerous and their influence on parameter estimates can be very destructive particularly a single large AO outlier many destroy the information content of the sample autocorrelation function or sample partial autocorrelation function. In modeling and analyzing time series the researcher must decide how to handle potential and known outliers. There are three ways to deal with outlying observation:

1. deleting
2. accommodation (robust estimates of model)
3. detecting , modeling and interpretation

Earlier, outlier were usually thrown out, but nowadays this is not usually recommended. In careful modeling, the outlying observation are replaced by some robust estimates. Robust estimation method are used in the modelling process: i.e. in the identification, estimation and diagnostic checking phases. It is important to know that one remarkable outlying observation can ruin OLS estimates. Robust regression estimation provides less biased parameter estimates and thus leads to residuals that enhance the visibility of possible outliers. (Kleiner et al., 1979)

It is well known that in the regression analysis, especially multivariate regression, the identification and detection of outlier is troublesome. In time series we encounter more difficulties due to the serial correlation between adjacent observa-

Table 4 : ESACF table

4.1	MA0	MA1	MA2	MA3	MA4	MA5		MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.96	0.93	0.90	0.87	0.84	0.81	AR0	X	X	X	X	X	X
AR1	-0.20	0.10	0.06	-0.02	0.03	-0.01	AR1	X	0	0	0	0	0
AR2	0.28	0.14	0.10	0.02	0.03	0.03	AR2	X	0	0	0	0	0
AR3	-0.47	0.32	0.15	-0.07	0.02	0.03	AR3	X	0	0	0	0	0
4.2	MA0	MA1	MA2	MA3	MA4	MA5		MA0	MA1	MA2	MA3	MA4	MA5
AR0	0.98	0.95	0.92	0.89	0.86	0.83	AR0	X	X	X	X	X	X
AR1	0.25	0.25	0.14	0.02	0.06	-0.02	AR1	X	X	0	0	0	0
AR2	-0.44	0.06	0.07	-0.08	0.06	-0.05	AR2	X	0	0	0	0	0
AR3	-0.35	0.19	0.06	-0.12	0.03	0.00	AR3	X	0	0	0	0	0

tions. In addition there are various types of outliers with different effects on observation. According to Durbin 1979 and Funke (1992), the Box-Jenkins univariate ARIMA modelling has been criticized for a lack of robustness. When the data are anticipated to contain outliers, first it is important to use robust methods. Thus if one is doing ARIMA modelling, the robust identification tools should be used first. The standard ESACF seems to be robust to some degree. Tsay (1986) remarked that the ESACF procedure may be robust to some degree if the number of outliers is small, the outliers are of moderate size and sample size is relatively large.

Due to common occurrence of outliers, the robustified EACF procedure is a reasonable tool for identifying ARIMA models. If there are outliers in the data and we use standard ESACF, we obtain very biased OLS estimates of the autoregression coefficients also in recursion estimation. Thus the ordinary OLS method in autoregression and the sample ACF in calculating autocorrelation of series must be replaced by their robust matches. For robustifying the OLS method we replace the minimizing function as following:

$$\sum (y_i - \sum x_{ij}\phi_j)^2 \rightarrow \sum \rho(y_i - \sum x_{ij}\phi_j)^2$$

$\rho(\cdot)$, the weight function is assumed to be convex, non-monotone and particularly first derivative should be continuous and bounded. By use this replacement, ESACF will be robustify. In robustifying the ESACF procedure we can first robustify the iterative AR(p) regression estimation and second implement a robust autocorrelation function for every iteration round. There are common regression estimators which are increas-

ing degree of robustness, One of them is M-estimator also there are different kinds of robust autocorrelation function like weighted autocorrelation function.

For the linear regression model $y_i = x_i' \beta + u_i$ the minimizing function for M-estimator of β is $\sum \rho(y_i - x_i' \beta / \sigma)$ where $\rho(\cdot)$ function is defined on R and σ^2 is variance of u . (more detail, Huber 1964, 1973).

Wacf is a robustified autocorrelation function in which each observation has its own weight, ω_i :

$$\hat{\rho}_\omega(k) = \frac{\hat{\gamma}_\omega(k)}{\hat{\gamma}_\omega(0)}$$

Where

$$\hat{\rho}_\omega(k) = \frac{\sum_{t=k+1}^n (z_{t-k} - \bar{z}_\omega)(z_t - \bar{z}_\omega) \omega_{t-k} \omega_t}{\sum_{t=k+1}^n \omega_{t-k} \omega_t}$$

And

$$\bar{z}_\omega = \frac{\sum_{t=1}^n z_t \omega_t}{\sum_{t=1}^n \omega_t}$$

(For more details Wang and Wei 1993).

Two additive outliers are located at $t_l = 139$, $\omega_l = -10$ and $t_l = 193$, $\omega_l = 10$. The contaminated series is displayed in following figure, in identification, the standard ESACF breaks down and indicate an ARMA (1,1) model (table 4.1), while the robust version, the M-estimator and the combination of OLS/Wacf indicate correctly an ARIMA(1,1,1) process (table 4.2).

CONCLUSION

As we mentioned earlier, there many different methods to detect the order of time series model. The Box-Jenkins methodology is not powerful to

identification the order of mixed models, because the performance of ACF and PACF are highly dependent to observation and there is a great sensitivity to the outliers. Due to these defects, we used the Tsay and Tiao (1984) method, with the presence of outliers, this method is not efficient. So with robust ESACF, We were able to identify the correct orders of models.

Stadnystska et al. (2008) compared the other methods like MINIC (minimum information criterion) and SCAN (smallest canonical correlation) with ESACF. The best result were 79% of correct identification for SCAN and 80% for ESACF. For some models and parameterization, the accuracy of SCAN and ESACF was disappointing. For autoregressive structures, MINIC achieved the best results, SCAN was superior to the other procedure for mixed models. For moving average processes, ESACF obtained the most correct selection. MINIC and SCAN had difficulty identifying moving average models. ESACF demonstrated low power in autoregressive cases. The method that described in this paper is better than the Box-Jenkins in term of optimality time.

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