Optimization
Iranian Journal of Optimization
Volume 7, Issue 2, 2015, 849-855
Research Paper
Online version is available on: www.ijo.iaurasht.ac.ir
Islamic Azad University
Rasht Branch
E-ISSN:2008-5427

A new Approach to Fuzzy Quantities Ordering Based on Distance Method and its Applications for Solving Fuzzy Linear Programming

Seyyed Hadi Nasseri*, Elham Darban Jafari and Roghayyeh Chameh

Department of Mathematics, University of Mazandaran, Babolsar, Iran

Received: 16 July 2015
Accepted: 19 September 2015

Abstract

Many ranking methods have been proposed so far. However, there is yet no method that can always give a satisfactory solution to every situation; some are counterintuitive, not discriminating; some use only the local information of fuzzy values; some produce different ranking for the same situation. For overcoming the above problems, we propose a new method for ranking fuzzy quantities based on the distance method. Then, an application of using fuzzy ordering in the fuzzy mathematical programming as well as fuzzy primal simplex algorithm is indicated. In particular, we emphasize that the fuzzy ordering will be useful when a decision maker needs to evaluate the optimality condition in any solving process.

Keywords:
fuzzy linear programming
fuzzy number
fuzzy ordering
simplex algorithm

***Correspondence E-mail**: nasseri@umz.ac.ir

INTRODUCTION

Ranking fuzzy numbers is a very important issue in fuzzy sets theory and applications, and has been extensively researched in the past. Many ranking methods have been proposed so far. However, there is yet no method that can always give a satisfactory solution to every situation; some are counterintuitive, not discriminating; some use only the local information of fuzzy values; some produce different ranking for the same situation. A significant number of ranking approaches have been suggested in the literature (Abbasbandy et al., 2006; Chen et al., 1992), (Chen et al., 2001; Fortemps et al.,1996), (Lee kwang et al., 1999; Liu et al., 2005), (Modarres et al., 2001; Nasseri et al., 2015), (Nasseri et al., 2010; Tran et al., 2002) and (Wang et al., 2008; Wang et al., 2001). Some of them have been reviewed and compared by Bortolan and Degani (1985), Chen and Hwang (2001), and Wang and et al. (2008, 2009). However, some of these methods have some advantages, but many of them have some shortages in the practical situations. For overcoming the above problems, we propose a new method for ranking fuzzy numbers by $D_{p,q}$ -distance. By introducing a maximum value crisp, minimum value crisp and $D_{p,q}$ -distance between two fuzzy numbers, we propose approach to ordering fuzzy numbers. Having reviewed the previous dictions, the researchers have two objectives in this study. Firstly they want to introduce a modified defuzzification of a fuzzy quantity. The second objective is applying the proposed method for ranking of fuzzy numbers. Here, a new approach for ordering fuzzy numbers is proposed. In particular, we will show that all reasonable properties which are established by Wang and Kerre (2001)are valid. Discussion and comparison of this work and other methods are carried out too. Finally, we give a new approach for fuzzy ordering to prepare a practical tool for solving the fuzzy linear programming problems which is established based on the ordering of the fuzzy quantities and intervals. However, the mentioned approach will be useful for solving multi-objective linear programming as well as discussed in (Mahdavi et al., 2007; 2009; Mishmast nehi et al., 2012; Nasseri et al., 2015) in uncertainty environments.

PRELIMINARIES AND BACKGROUNDS

The basic definitions and concepts of the fuzzy sets theory are given as follows which is taken from (Cheng,1998) and (Kaufmann et al,1985).

Definition 2.1: Let X be a universe set. A fuzzy set A of X is defined by a membership function $\mu A (x): R \rightarrow [0,1]$, where $\mu A (x), \forall x \in X$, indicates the degree of x in A.

Remark 2.1: In throughout the paper we assume that $X=R$.

Definition 2.2: A fuzzy subset A of universe set X is normal iff $sup_{x \in X}\, \mu A (x)=1$, where X is the universe set.

Definition 2.3: A fuzzy subset A of universe set X is convex iff $\mu_A (\lambda x_1+(1-\lambda)x_2) \geq min\{\mu_A (x_1),\mu_A (x_2)\};x_1,x_2 \in X,\lambda \in [0,1]$.

Definition 2.4: A fuzzy subset A is a fuzzy number iff A is normal and convex on X.

zzy number A is rship function

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x < a_2, \\ 1, & a_2 \leq x < a_3, \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x < a_4, \\ 0, & otherwise, \end{cases} \quad (2\text{-}1)$$

which can be denoted as a quartet (a_1,a_2,a_3,a_4). In these above situations $a1,a2,a3$ and $a4$, if $a2=a3$, A becomes a triangular fuzzy number.

Definition 2.6: An extended fuzzy number A is described as any fuzzy subset of the universe set X with membership function μ_A defined as follows:

(a) $\mu_A(x)$ is a continuous mapping from X to the closed interval [0,1].

(b) $\mu_A(x)=0$, for all $x \in (-\infty,a_1]$.

(c) μ_A is strictly increasing on $[a_1,a_2]$.

(d) $\mu_A(x)=1$, for all $x \in [a_2,a_3]$.

(e) μ_A is strictly decreasing on $[a_3,a_4]$.

(f) $\mu_A(x)=0$, for all $x \in [a_4,+\infty)$.

where a_1,a_2,a_3 and a_4 are real numbers. If $a_1=a_2=a_3=a_4$, A becomes a crisp real number.

Remark 2.2: Throughout the paper, we let $\tilde{O}=(0,0,0,0)$ as the zero trapezoidal fuzzy number.

Definition 2.7: The *α-cut* of a fuzzy number A, where $0<\alpha \leq 1$ is a set defined as

$A\alpha=\{x \in R | \mu_A(x) \geq \alpha\}$.

According to the definition of a fuzzy number it is seen once that every *α-cut* of a fuzzy number is a closed interval. Hence, we have $A_\alpha=[A^-_\alpha, A^+_\alpha]$, where

$$A^-_\alpha=inf\{x \in R | \mu_A(x) \geq \alpha\}, \quad (2\text{-}2)$$

$$A^+_\alpha=sup\{x \in R | \mu_A(x) \geq \alpha\}. \quad (2\text{-}3)$$

A set of all fuzzy numbers on real line will be denoted by F(R), and this article recalls that core $A=\{x\in R|\ \mu_A(x)=1\}$.

Definition 2.8: *A* $D_{p,q}$ -distance, indexed by parameters $1\leq p\leq\infty, 0\leq q\leq 1$, between two fuzzy numbers A and B is a nonnegative function on F(R)×F(R) give as follows:

$$D_{p,q}(A,B)=\begin{cases}\left[(1-q)\int_0^1 |A_\alpha^- - B_\alpha^-|^p d\alpha + q\int_0^1 |A_\alpha^+ - B_\alpha^+|^p d\alpha\right]^{\frac{1}{p}}, if\ \ p<\infty, \\ (1-q)\ \sup_{0<\alpha\leq 1}(|A_\alpha^- - B_\alpha^-|^p) + q\ \inf_{0<\alpha\leq 1}(|A_\alpha^+ - B_\alpha^+|^p),\ if\ \ p=\infty.\end{cases} \quad (2\text{-}4)$$

In this paper, we suppose that p=2 and q=1/2. Hence:

$$\left[D_{2,\frac{1}{2}}(A,B)\right]^2=\frac{1}{2}\left(\int_0^1 |A_\alpha^- - B_\alpha^-|^2 d\alpha + \int_0^1 |A_\alpha^+ - B_\alpha^+|^2 d\alpha\right) \quad (2\text{-}5)$$

Definition 2.9: Let $A=(a_1,a_2,a_3,a_4)$ and $B=(b_1, b_2,b_3,b_4)$ be two trapezoidal fuzzy numbers, and

$A_\alpha=[A^-_\alpha,A^+_\alpha]=[(1-\alpha)\ a_1+\alpha a_2,\ \alpha a_3+(1-\alpha)\ a_4]$,

$B_\alpha=[B^-_\alpha,B^+_\alpha]=\ [(1-\alpha)\ b_1+\alpha b_2,\alpha b_3+(1-\alpha)\ b_4]$.

Define the $D_{2,1/2}$-distance for two trapezoidal fuzzy numbers A and B on F(R) as follows:

$$\left[D_{2,\frac{1}{2}}(A,B)\right]^2=\frac{1}{6}\left[\sum_{i=1}^4 (b_i-a_i)^2+\sum_{i\in\{1,3\}}(b_i-a_i)(b_{i+1}-a_{i+1})\right]. \quad (2\text{-}6)$$

Let $A=(a_1,a_2,a_3,a_4)$ and $B=(b_1,b_2,b_3,b_4)$ be two trapezoidal fuzzy numbers and x∈R. Define:

$x\geq 0, xA=(xa_1,xa_2,xa_3,xa_4)$,

$x<0, xA=(xa_4,xa_3,xa_2,xa_1)$,

$A+B=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4)$.

CONSTRUCTION OF A NEW METHOD FOR ORDERING OF FUZZY NUMBER

Here, we give a new approach to order the fuzzy numbers that will be useful as a tool in a sub-algorithm for establishing some numerical process to solve fuzzy linear programming problems, interval linear programming problems, multi-objective fuzzy linear programming and many other convenient models which are appeared in the literature See in (Mahdavi et al., 2007,2009; Mishmast nehi et al., 2012; Nasseri , 2015; Nasseri et al., 2015).

Definition 3.1: First define a minimum crisp value $\mu_{Fmin}(x)$ and a maximum crisp value $\mu_{Fmax}(x)$ to be the benchmark, where their characteristic functions $\mu_{Fmin}(x)$ and $\mu_{Fmax}(x)$ are as follows:

$$\mu_{\mathcal{F}_{min}}(x)=\begin{cases}1, & x=\mathcal{F}_{min}, \\ 0, & x\neq\mathcal{F}_{min},\end{cases} \quad (3\text{-}1)$$

and

$$\mu_{\mathcal{F}_{max}}(x)=\begin{cases}1, & x=\mathcal{F}_{max}, \\ 0, & x\neq\mathcal{F}_{max}.\end{cases} \quad (3\text{-}2)$$

Now assume that *n* fuzzy numbers $A_1, A_2, \dots, A_n$ on F(R) are given, then the minimum crisp value F_{min} and the maximum crisp value F_{max} are defined as:

$$\mathcal{F}_{min}=\inf\bigcup_{i=1}^n supp(A_i), \quad (3\text{-}3)$$

$$\mathcal{F}_{max}=\sup\bigcup_{i=1}^n supp(A_i). \quad (3\text{-}4)$$

Let $A_i,A_j\in F(R)(\forall i\neq j)$ be two arbitrary fuzzy numbers. Define the rank of A_i and A_j by $D_{2,1/2}$ on F(R) as follows:

$(1) A_i\succ A_j$ by $D_{2,1/2}\Leftrightarrow D_{2,1/2}(A_i,F_{min})>D_{2,1/2}(A_j,F_{min})$,

$(2) A_i\sim A_j$ by $D_{2,1/2}\Leftrightarrow D_{2,1/2}(A_i,F_{min})=D_{2,1/2}(A_j,F_{min})$,

$(3) A_i\prec A_j$ by $D_{2,1/2}\Leftrightarrow D_{2,1/2}(A_i,F_{min})<D_{2,1/2}(A_j,F_{min})$. (3-5)

Also, we formulate the order $\succcurlyeq$ and $\preccurlyeq$ as $Ai\succcurlyeq Aj$ if and only if $A_i\succ A_j$ or $A_i\sim A_j$, $A_i\preccurlyeq A_j$ if and only if $A_i\prec A_j$ or $A_i\sim A_j$.

Moreover, we may use the F_{max} index as well as $F_{min,}$ and then for two arbitrary fuzzy numbers $A_i,\ A_j\in F(R)(\forall i\neq j)$, we may define the rank of A_i and A_j by $D_{2,1/2}$ on F(R) as follows:

$A_i\succ A_j$ by $D_{2,1/2}\Leftrightarrow D_{2,1/2}\ A_i,F_{max}<D_{2,1/2}(A_j,F_{max})$,

$A_i\sim A_j$ by $D_{2,1/2}\Leftrightarrow D_{2,1/2}(A_i,F_{max})=D_{2,1/2}(A_j,F_{max})$,

$A_i\prec A_j$ by $D_{2,1/2}\Leftrightarrow D_{2,1/2}(A_i,F_{max})>D_{2,1/2}(A_j,F_{max})$. (3-6)

Now we are a place to illustrate the mentioned approach.

Example 3.1: Consider the following fuzzy numbers in the trapezoidal format which is shown with four parameters:

A_1= (2,3,3,6),A_2=(3,4,4,5) and A_3=(2,4,6,7).

The fuzzy numbers and the minimum and maximum crisp values are illustrated in Fig. 1.

Hence, according to Eq. 2-2, 2-3, 3-3 and 3-4, the parameters F_{min}, F_{max} and the inverse functions obtain as follows:

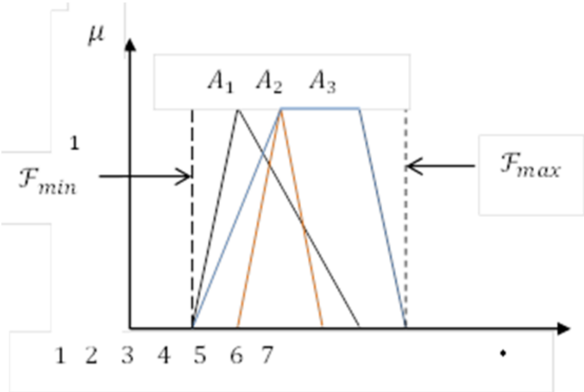

Fig.1. Fuzzy numbers A_1, A_2, A_3, F_{max} and F_{min}

$$\mathcal{F}_{min} = inf \bigcup_{i=1}^{3} supp(A_i) = 2,$$

and

$$\mathcal{F}_{max} = sup \bigcup_{i=1}^{3} supp(A_i) = 7,$$

and

$$g_{\min(x)} = \begin{cases} g^{-}_{\min(x)} = 2, \\ g^{+}_{\min(x)} = 2, \end{cases}$$

and

$$g_{max}(x) = \begin{cases} g^{-}_{max}(x) = 7, \\ g^{+}_{max}(x) = 7. \end{cases}$$

According to Eq. 2-6, we can get the $D_{2,1/2}$–distance values the between maximum crisp value and fuzzy numbers A_1, A_2 and A_3, which are equal to 13.66,9.33 and 8.33, respectively. Thus, by Eq. 3-6 it is follows that:

$A_3 \succ A_2 \succ A_1$.

In the below, the membership function of fuzzy numbers A_1, A_2 and A_3 and their inverse functions of them are given:

$$A_1: \qquad \mu_{A_1}(x) = \begin{cases} x-2, & 2 \le x \le 3, \\ \frac{6-x}{3}, & 3 \le x \le 6, \\ 0, & otherwise, \end{cases}$$

$$g_{A_1}(x) = \begin{cases} A_1^{-}(x) = x+2, \\ A_1^{+}(x) = 6-3x. \end{cases}$$

$$A_2: \qquad \mu_{A_2}(x) = \begin{cases} x-3, & 3 \le x \le 4, \\ 5-x, & 4 \le x \le 5, \\ 0, & otherwise, \end{cases}$$

$$g_{A_2}(x) = \begin{cases} A_2^{-}(x) = x+3, \\ A_2^{+}(x) = 5-x. \end{cases}$$

$$A_3: \qquad \mu_{A_3}(x) = \begin{cases} \frac{x-2}{2}, & 2 \le x \le 4, \\ 1, & 4 \le x \le 6, \\ 7-x, & 6 \le x \le 7, \\ 0, & otherwise, \end{cases}$$

$$g_{A_3}(x) = \begin{cases} A_3^{-}(x) = 2x+2, \\ A_3^{+}(x) = 7-x. \end{cases}$$

Now we are going to verify that the reasonable axioms which are established by Wang and Kerre (2001) are also valid for the proposed approach. Let S be a set of fuzzy quantities for which the method $D_{2,1/2}$–distance can be applied, and $\mathcal{A}$ and $\mathcal{A}'$ finite subset of *S*. The statement "two elements *A* and *B* in $\mathcal{A}$ satisfy that *A* has a higher rank than *B* when $D_{2,1/2}$–distance is applied to the fuzzy quantities in $\mathcal{A}$ "will be written as" $A \succ B$ by $D_{2,1/2}$ on $\mathcal{A}$"."$A \sim B$ by $D_{2,1/2}$ on $\mathcal{A}$" and "$A \succcurlyeq B$ by $D_{2,1/2}$ on $\mathcal{A}$" are similarly interpreted.

The following axioms show the reasonable properties of the ordering approach $D_{2,1/2}$–distance.

A_1. For $A \in \mathcal{A}$, $A \succcurlyeq A$ by $D_{2,1/2}$ on $\mathcal{A}$.

A_2. For $(A,B) \in \mathcal{A}^2$, $A \succcurlyeq B$ and $B \succcurlyeq A$ by $D_{2,1/2}$ on $\mathcal{A}$, we should have $A \sim B$ by $D_{2,1/2}$ on $\mathcal{A}$.

A_3. For $(A,B,C) \in \mathcal{A}^3$, $A \succcurlyeq B$ and $B \succcurlyeq C$ by $D_{2,1/2}$ on A, we should have $A \succcurlyeq C$ by $D_{2,1/2}$ on $\mathcal{A}$.

A_4. For $(A,B) \in \mathcal{A}^2$, inf supp(A)>sup supp(B), we should have $A \succcurlyeq B$ by $D_{2,1/2}$ on $\mathcal{A}$.

A_4'. For $(A,B) \in \mathcal{A}^2$,inf supp(A)>sup supp(B), we should have $A \succ B$ by $D_{2,1/2}$ on $\mathcal{A}$.

A_5. Let $(A,B) \in (\mathcal{A} \cap \mathcal{A}')^2$. We obtain the ranking order $A \succcurlyeq B$ by $D_{2,1/2}$ on $\mathcal{A}'$ if and only if $A \succcurlyeq B$ by $D_{2,1/2}$ on $\mathcal{A}$.

A_6. Let A,B,A+C and B+C be elements of S. If $A \succcurlyeq B$ by $D_{2,1/2}$ on {A,B}, then $A+C \succcurlyeq B+C$ by $D_{2,1/2}$ on {A+C,A+B}.

A_6'. Let A,B,A+C and B+C be elements of S and $C=\varnothing$. If $A \succ B$ by $D_{2,1/2}$ on {A,B}, then $A+C \succ B+C$ by $D_{2,1/2}$ on {A+C,A+B}.

Remark 3.1: From Definition 3.1, we clearly if $A \succcurlyeq B$, then $-B \succcurlyeq -A$.

In the next section, we will give an application of the emphasized discussion in the previous sections to motivate the given study.

AN APPLICATION AND MOTIVATION

In this section, an application of the concept of ordering on fuzzy quantities is given to support the motivation of our discussion. For achieving a solution for the following fuzzy mathematical model, it is necessary to solve the model using a numerical algorithm such as fuzzy primal or/and dual simplex algorithms as well as have been suggested in (Mahdavi et al., 2007; 2009; Mishmast nehi et al., 2012; Nasseri et al., 2015) based on the formulated problems in the real life.

Example 4.1. Assume that a company makes two products. Product P_1 has a profit of around \$40 per unit and product P_2 has a profit of around \$30 per unit. Each unit of P_1 requires twice as many labor hours as each available labor hours are somewhat close to 500 h per day, and may possibly be changed due to special arrangements for overtime work. The supply of material is almost 400 units of both products, P_1 and P_2, per day, but may possibly be changed according to past experience.The problem is, how many units of products P_1 and P_2 should be made per day to maximize the total profit? Let $\tilde{x}1$ and $\tilde{x}2$ denote the number of units of products P_1 and P_2 made in one day, respectively. Then, the problem can be formulated as the following fuzzy linear programming problem:

$$
\begin{aligned}
max \quad & \tilde{z} \approx \widetilde{40}\tilde{x}_1 + \widetilde{30}\,\tilde{x}_2 \\
s.t. \quad & \begin{cases} \tilde{x}_1 + \tilde{x}_2 \preccurlyeq \widetilde{400} \\ 2\tilde{x}_1 + \tilde{x}_2 \preccurlyeq \widetilde{500} \\ \tilde{x}_1 \text{ and } \tilde{x}_2 \succcurlyeq \tilde{0} \end{cases}
\end{aligned}
$$

For solving this problem one may use a fuzzy primal simplex algorithm and therefore the first tableau of this algorithm can be made as follows:

Basis	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	R.H.S.
$\tilde{z}$	$\widetilde{40}$	$\widetilde{30}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{x}_3$	1	1	1	0	$\widetilde{400}$
$\tilde{x}_4$	2	1	0	1	$\widetilde{500}$

For evaluating the optimality condition of the current fuzzy basic feasible solution and all solutions in the next iterations, if the solution is not optimal, it must be calculated by the following formula:

$$\tilde{y}_{0j} = \tilde{c}_B B^{-1} a_j - \tilde{c}_j$$

where $\tilde{c}_B$ and $\tilde{c}_j$ are respectively the basic variables coefficients vector and the coefficient of $\tilde{x}_j$ in objective function, and also B^{-1} and a_j are respectively the inverse of the basis matrix and the coefficients vector of $\tilde{x}_j$ in the constraints of the model. Since here, solving this model is not our main discussion; hence we omit the solving process. We again emphasize that the mentioned problem which is formulated as a fuzzy mathematical programming problem is one of many problems which are modeled in the real world and may be needed to use an algorithm for solving the model where the fuzzy ordering plays a key role.

CONCLUSIONS

In this paper, we established a new method for ranking of fuzzy numbers. We saw that the proposed method can effectively rank various fuzzy numbers. Also, from experimental results, the new method with some advantages: (a) fit all kind of ranking fuzzy number, and (b) correct Kerre's concept (regardless of its relative location on the X-axis). Therefore, we suggested the proposed approach as a sub algorithm for solving some common practical models such as mathematical programming and in particular fuzzy linear programming models where ordering of fuzzy numbers for solving the problems is so important. Finally, we emphasize that the mentioned idea can be used for solving the classical mathematical problems such as Transportation, Assignment, and etc., as some convenient kinds of linear programming models in the real life which are formulated in uncertainty conditions.

REFERENCE

Abbasbandy, S., & Asady, B. (2006). Ranking of fuzzy numbers by sign distance. *Information Sciences, 176*(16), 2405-2416.

Asady, B., & Zendehnam, A. (2007). Ranking fuzzy numbers by distance minimization. *Applied Mathematical Modelling, 31*(11), 2589-2598.

Bortolan, G., & Degani, R. (1985). A review of some methods for ranking fuzzy subsets. *Fuzzy sets and Systems, 15*(1), 1-19.

Chen, S. J., & Hwang, C. L. (1992). Fuzzy multiple attribute decision making methods. In *Fuzzy Multiple Attribute Decision Making* (pp. 289-486). Springer Berlin Heidelberg.

Chen, L. H., & Lu, H. W. (2001). An approximate approach for ranking fuzzy numbers based on left and right dominance. *Computers & Mathematics with Applications, 41*(12), 1589-1602.

Cheng, C. H. (1998). A new approach for ranking fuzzy numbers by distance method. *Fuzzy sets and systems, 95*(3), 307-317.

Chu, T. C., & Tsao, C. T. (2002). Ranking fuzzy numbers with an area between the centroid point and original point. *Computers & Mathematics with Applications, 43*(1-2), 111-117.

Deng, Y., Zhenfu, Z., & Qi, L. (2006). Ranking fuzzy numbers with an area method using radius of gyration. *Computers & Mathematics with Applications, 51*(6-7), 1127-1136.

Fortemps, P., & Roubens, M. (1996). Ranking and defuzzification methods based on area compensation. *Fuzzy sets and systems, 82*(3), 319-330.

Kaufmann, A., & Gupta, M. M. (1985). Introduction to fuzzy arithmetic. 1985. *Reinhold, New York*.

Lee-Kwang, H., & Lee, J. H. (1999). A method for ranking fuzzy numbers and its application to decision-making. *IEEE Transactions on Fuzzy Systems, 7*(6), 677-685.

Liou, T.S., & Wang, M.J.J. (1992). Ranking fuzzy numbers with integral value. *Fuzzy sets and systems, 50*(3), 247-255.

Liu, X. W., & Han, S. L. (2005). Ranking fuzzy numbers with preference weighting function expectations. *Computers & Mathematics with Applications, 49*(11), 1731-1753.

Mahdavi Amiri, N. ,& Nasseri, S.H. (2007). Duality results and a dual simplex method for linear programming with trapezoidal fuzzy variables. *Fuzzy Sets and Systems, 158*(17) ,1961-1978.

Mahdavi Amiri, N., Nasseri, S.H., & Yazdani, A. (2009).Fuzzy primal simplex algorithms for solving fuzzy linear programming problems. *Iranian Journal of Operational Research, 1*(2), 68-84.

Mishmast Nehi, H., & Hajmohamadi, H. (2012). A ranking function method for solving fuzzy multi-objective linear programming problem. *Annals of Fuzzy Mathematics and Informatics, 3*(1) ,31- 38.

Modarres, M., & Sadi-Nezhad, S. (2001). Ranking fuzzy numbers by preference ratio. *Fuzzy Sets and Systems, 118*(3), 429-436.

Nasseri, S.H.(2015). Ranking trapezoidal fuzzy numbers by using Hadi method. *Australian Journal of Basic and Applied Sciences 4*(8) 3519-3525.

Nasseri , S.H., & Khazaee Kohpar, O. (2015). Pareto-optimal solutions in multi-objective linear programming with fuzzy numbers. *Annals of Fuzzy Mathematics and Informatics, 9*(5), 823-833.

Nasseri, S.H., & Mizuno, S. (2010). A new method for ordering triangular fuzzy numbers. *Iranian Journal of Optimization, 6*(1), 603-613.

Nasseri, S.H., & Sohrabi, M. (2010). Hadi's method and its advantage in ranking fuzzy numbers. *Australian Journal of Basic Applied Sciences, 4*(10), 4630-4637.

Nasseri, S.H., Taleshian, F., Alizadeh, Z., & Vahidi, J. (2012). A new method for ordering LR fuzzy number. *The Journal of Mathematics and Computer Science, 4*(3) ,283-294.

Nasseri, S.H., Zadeh, M.M., Kardost, M., & Behmanesh, E. (2013). Ranking fuzzy quantities based on the angle of the reference functions. *Applied Mathematical Modelling, 37*(22), 9230-9241.

Sun, H., & Wu, J. (2006). A new approach for ranking fuzzy numbers based on fuzzy simulation analysis method. *Applied Mathematics and Computation, 174*(1), 755-767.

Tran, L., & Duckstein, L. (2002). Comparison of fuzzy numbers using a fuzzy distance measure. *Fuzzy sets and Systems, 130*(3), 331-341.

Wang, X., & Kerre, E. E. (2001). Reasonable properties for the ordering of fuzzy quantities (I). *Fuzzy sets and systems, 118*(3), 375-385.

Wang, Y. J., & Lee, H. S. (2008). The revised method of ranking fuzzy numbers with an area between the centroid and original points. *Computers & Mathematics with Applications, 55*(9), 2033-2042.

Wang, Z. X., Liu, Y. J., Fan, Z. P., & Feng, B. (2009). Ranking L–R fuzzy number based on deviation degree. *Information Sciences, 179*(13), 2070-2077.

Yao, J. S., & Wu, K. (2000). Ranking fuzzy numbers based on decomposition principle and signed distance. *Fuzzy sets and Systems, 116*(2), 275-288.