Merging DMUs Based on the Idea Inverse DEA

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Abstract
In this paper, we propose a novel method using multiple-objective programming problems to answer the following question: if among a group of decision making units (DMUs), a subset of DMUs are required to merge and form a new DMU with specific input/output levels and a predefined efficiency target, how much should be the outputs/inputs of the merged DMU? This question answered according to the concept of inverse DEA. Sufficient conditions are established for input/output-estimation of the merged DMU using multiple-objective programming problems. A numerical example with real data is presented to illustrate the goals of this paper.

Keywords:
Data Envelopment Analysis (DEA)
Efficiency
Inverse DEA
Multiple-Objective Programming (MOP)
Merging decision making units (DMUs)

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INTRODUCTION

One of the efficient mathematical tools for evaluating the performance of the DMUs is the DEA technique which is proposed by Charnes et al. (1978) and developed by many scholars, see e.g. (Banker et al., 1984; Cook & Seiford, 2009; Emrouznejad & Tavana, 2014) for some reviews. DEA proposes a technique to estimate the relative efficiency of a group of DMUs with multiple inputs and outputs such as bank branches.

Zhang and Cui (1999) presented the idea of inverse DEA. In fact, inverse DEA proposed to answer this question: among a set of DMUs, if decision maker changes inputs/outputs a particular DMU under preserving the efficiency level, how much should the outputs/inputs of the DMU change? This question answered using multiple-objective linear Programming (MOLP) problems by Hadi-Vencheh et al. (2006), though its question in a special case (increase inputs/outputs) was studied by Wei et al. (2000). This question studied under improving efficiency index by Jahanshahloo et al. (2004a; 2004b). In the inverse DEA filed, the problem of simultaneous estimation of input-output levels are proposed by Jahanshahloo et al. (2014) and Ghabadi (2017). Inverse DEA has been used and developed by many scholars, see e.g. (Gattoufi et al., 2014; Ghabadi et al., 2014; Jahanshahloo et al., 2015).

Inverse DEA is important from both theoretical and practically points of view, because this technique can be used in the different framework, including preserve (improve) efficiency values (Lertworasirikul et al., 2011), resource allocation (Hadi-Vencheh et al., 2006), and firms restructuring (Amin et al., In Press). In this direction, the idea of the inverse DEA used to merging the banks by Gattoufi et al. (2014). In fact, the inverse DEA applied to answer the following question:

Question: If among a set of DMUs, a subset of DMUs are required to merge and form a new DMU with specific input/output levels and a predetermined efficiency target, how much should be the outputs/inputs of the merged DMU?

A technique was proposed to answer Question using mathematical programming by Gattoufi et al. (2014).

In this paper, we proposed a novel method to answer the above question using multiple-objective programming problems. Sufficient conditions are proposed for input/output-estimation of the merged DMU using multiple-objective programming problems. This method, unlike other proposed method (Gattoufi et al., 2014), decreases the number of the variables of the model strongly, and this decreases the computational complexity. A numerical example with real data is provided to illustrate the goals of this paper.

The structure of the paper is as follows: Section 2 gives some preliminaries from DEA. The problem of the merging DMUs investigated in section 3. This section proposes a new method to solving the problem of the merging DMUs. Sufficient conditions are proposed for input/output-estimation of the merged DMU. An example with real data provided in section 4. Section 5 gives a brief conclusion and directions for future research.

PRELIMINARIES FROM DEA

Suppose that there is a set of n DMUs, \{\text{DMU}_j\;:\; j=1,\ldots,n\} which \text{DMU}_j (j=1,\ldots,n) uses multiple positive inputs \textbf{x}_j=(x_{1j}, x_{2j}, \ldots, x_{mj}) to produce multiple positive outputs \textbf{y}_j=(y_{1j}, y_{2j}, \ldots, y_{sj}). The following model is considered to estimate the relative efficiency of the unit under assessment \text{DMU}_o, o={1, 2, \ldots, n}, as follows:

\[
\begin{align*}
\sigma_o^* &= \min \theta \\
\text{subject to} & \sum_{j \in \Omega} \lambda_j x_{ij} \leq \theta x_{io}, & i=1, \ldots, m, \\
& \sum_{j \in \Omega} \lambda_j y_{ij} \geq \theta y_{io}, & r=1, \ldots, s, \\
& \lambda \in \Omega,
\end{align*}
\]

where:

\[
\Omega = \left\{ \lambda = (\lambda_1, \ldots, \lambda_n) : \sigma_1 \left( \sum_{j \in \Omega} \lambda_j - \sigma_2 \right) \geq 0, \lambda_j \geq 0, j=1, \ldots, n \right\}.
\]

Here \(\sigma_1, \sigma_2,\) and \(\sigma_3\) are parameters with 0-1 values. It is obvious that:

(i) If \(\sigma = 0\), then model (1) is under a constant returns to scale (CRS) assumption of the production technology.

(ii) If \(\sigma = 1\) and \(\sigma = 1\), then model (1) is under a variable returns to scale (VRS) assumption of the production technology.
(iii) If $\sigma_1 = \sigma_2 = 1$ and $\sigma_3 = 0$, then model (1) is under a non-increasing returns to scale (NIRS) assumption of the production technology.

(iv) If $\sigma_1 = \sigma_2 = \sigma_3 = 1$, then model (1) is under a non-decreasing returns to scale (NDRS) assumption of the production technology.

$\theta_{o}^*$ in Model (1) is called the input-oriented efficiency score of $DMU_o$. It is obvious that $\theta_{o}^* \leq 1$. If $\theta_{o}^* = 1$, then $DMU_o$ is called input-oriented weakly efficient.

Model (1) is called an input-oriented DEA model. The output-oriented version of this model is as follows:

$$\phi_{o}^* = \max \phi$$

$$s.t. \quad \sum_{j \in \Pi} \lambda_j y_{ij} - x_{ij}, \quad i = 1, 2, ..., m,$$

$$\sum_{j \in \Pi} \lambda_j y_{rj} \geq \theta_{o} y_{rM}, \quad r = 1, 2, ..., s,$$

$$\lambda \in \Omega.$$

$\phi_{o}^*$ in Model (2) is called the output-oriented efficiency score of $DMU_o$. It is obvious that $\phi_{o}^* \leq 1$. In addition, $DMU_o$ is called output-oriented weakly efficient if $\phi_{o}^* = 1$.

Remark 1.2 If $\theta_{o}^* < 1$ or $\phi_{o}^* > 1$, which is the $DMU_o$ is inefficient, then $DMU_o$ can be presented by the efficient DMUs. Therefore, the corresponding $\lambda_{o}$ will be zero in optimality ($\lambda_{o}^* = 0$).

MERGING DMUs

In this section a new method suggested for merger DMUs using the Inverse DEA approach and MOP problems. This method allows determining the levels of inputs and outputs for a single merged DMU, following a merger between at least two DMUs.

Let us to assume that there is a set of $n$ DMUs in which $DMU_j, j \in J = \{1, 2, ..., n\}$, uses $m$ inputs $x_{ij}$ in order to produce $s$ outputs $y_{rj}$, for all $i = 1, 2, ..., m$ and $r = 1, 2, ..., s$. Assume that the all inputs and outputs are positive. Suppose that the set DMUs, $J = \{1, 2, ..., n\}$, is divided into two subsets $\Pi$ and $\Gamma$, where $\Pi \cap \Gamma = J$, $\Pi \cup \Gamma = J$, and $\Pi \cap \Gamma = \phi$. Assume that DMUs, $j \in \Pi$ are merged and are looking to create a new DMU, namely $DMU_{M}$. In addition, suppose that $\theta_{M}^*$ is a predetermined target for efficiency of the $DMU_{M}$.

Initially, in order to present suitable patterns to the decision maker to estimate input/output vector $DMU_{M}$, the following question is considered:

**Question 1.** If $DMU_j, j \in \Pi$ are required to merge and form a new unit ($DMU_{M}$), in which output vector and predetermined efficiency target of $DMU_{M}$ are $Y_{M} = \sum_{j \in J} Y_{j}$ and $\theta_{M}^*$, respectively, how much should be input vector of this new DMU?

Note that $DMU_{M}$ keeps the amount of outputs of all $DMU_j, j \in \Pi$, and looking to find the minimum amount of inputs of these DMUs in order to reach the pre-defined target level. The aim of the Question 1 is estimating the input vector $X_{M}$ provided that the efficiency index of $DMU_{M}$ is $\theta_{M}^*$. In other words, the optimal value of the following model is equal $\theta_{M}^*$.

$$\theta^* = \min \theta$$

$$s.t. \quad \sum_{j \in J} \lambda_j x_{ij} + \lambda_{M} x_{M} \leq \theta_{M} x_{M}, \quad i = 1, 2, ..., m,$$

$$\sum_{j \in J} \lambda_j y_{rj} + \lambda_{M} y_{rM} \geq \theta_{M} y_{rM}, \quad r = 1, 2, ..., s,$$

$$\lambda \in \Omega.$$

where $y_{rM} = \sum_{j \in J} y_{rj}$, $r = 1, 2, ..., s$ and:

$$\Omega = \left\{ \lambda = (\lambda_{j} : j \in \Gamma, \lambda_{M}) \in \mathbb{R}_{+}^{n+1} \right\},$$

$$\sigma \left( \sum_{j \in J} \lambda_{j} + \lambda_{M} + \sigma_{M} (-1)^{n+1} \right) = \sigma, \quad \nu \geq 0.$$

To solve Question 1, the following multiple objective non-linear programming (MONLP) problem is considered:

$$\min \left\{ \sum_{j \in J} \sigma_{j} : i = 1, 2, ..., m \right\}$$

$$s.t. \quad \sum_{j \in J} \lambda_j x_{ij} + \lambda_{M} x_{M} \leq \theta_{M} x_{M}, \quad i = 1, 2, ..., m,$$

$$\sum_{j \in J} \lambda_j y_{rj} + \lambda_{M} y_{rM} \geq \theta_{M} y_{rM}, \quad r = 1, 2, ..., s, \quad \lambda \in \Omega,$$

$$0 \leq \sum_{j \in J} \sigma_{j} \leq \sum_{j \in J} y_{ij}, \quad i = 1, 2, ..., m.$$
In the real world, the most common consolidations happen between DMUs to improve their respective performances. Therefore, we can assume that the merging DMUs are inefficient. It is obvious that if $\hat{\theta}_m < 1$ or even $\hat{\theta}_m = 1$, then DMU $M$ can be presented by the other efficient DMUs, and so the corresponding $\lambda_M$ will be zero in optimality ($\lambda_M^* = 0$). According to the above discuss, non-linear model (5) could be converted to the following linear model:

\[
\min \sum_{i=1}^{m} \sum_{j \in \Pi} \alpha_{ij} \xi_{ij} \\
\text{subject to} \sum_{i=1}^{m} \sum_{j \in \Pi} \alpha_{ij} \xi_{ij} \leq \bar{\theta} \left( \sum_{j \in \Pi} \gamma_j \right), \quad i = 1, 2, ..., m, \\
\sum_{i=1}^{m} \sum_{j \in \Pi} \alpha_{ij} \xi_{ij} \geq \sum_{j \in \Pi} \gamma_j, \quad r = 1, 2, ..., z, \\
0 \leq \sum_{i=1}^{m} \alpha_{ij} \xi_{ij} \leq \sum_{j \in \Pi} \gamma_j, \quad i = 1, 2, ..., m, \\
\lambda \in \Omega.
\]

Remark 1.3 In Model (6), $\sum_{i \in \Pi} \alpha_{ij}$ is unknown. Considering $\alpha_i M = \sum_{j \in \Pi} \alpha_{ij}$, as a new variable in Model (6), the number of variables is strongly reduced, and so computational complexity is reduced.

The following theorem shows how Model (6) can be used for input estimation of DMU $M$.

**Theorem 1.3** Let $DMU_j, \forall j \in \Pi$ be inefficient. If $A = (\lambda_i^* : j \in \Pi, \lambda_i M^* = 0)$ is an optimal solution to model (6), then efficiency score $DMU_M$ with the input vector $x_M = \sum_{j \in \Pi} \alpha_{ij}^*$ and output vector $y_M = \sum_{j \in \Pi} y_j$ is equal to $\hat{\theta}_M$.

**Proof.** It is obvious that $A = (\lambda_i^* : j \in \Pi, \lambda_i M^* = 0)$ is a feasible solution to model (3). Therefore, $\hat{\theta}_M < 0$. By contradiction assume that $A = (\lambda_i^* : j \in \Pi, \lambda_i M^* = 0)$ is an optimal solution to model (4) such that, $\hat{\theta}_M < 0$. Feasibility of $\nabla = (\lambda_i^*, \sum_{j \in \Pi} \alpha_{ij}^*)$ for LP (6), implies

\[
\sum_{i=1}^{m} \sum_{j \in \Pi} \alpha_{ij} \xi_{ij} \leq \bar{\theta} \left( \sum_{j \in \Pi} \gamma_j \right), \quad i = 1, 2, ..., m, \\
\sum_{i=1}^{m} \sum_{j \in \Pi} \alpha_{ij} \xi_{ij} \geq \sum_{j \in \Pi} \gamma_j, \quad r = 1, 2, ..., z, \\
0 \leq \sum_{i=1}^{m} \alpha_{ij} \xi_{ij} \leq \sum_{j \in \Pi} \gamma_j, \quad i = 1, 2, ..., m, \\
\lambda \in \Omega.
\]

By Eqs 7-10 and feasibility of $A = (\lambda_i^* : j \in \Pi, \lambda_i M^* = 0)$ for problem (3), we have:

\[
\sum_{i=1}^{m} \sum_{j \in \Pi} \alpha_{ij} \xi_{ij} \leq \bar{\theta} \left( \sum_{j \in \Pi} \gamma_j \right) = \bar{\theta} \left( \sum_{j \in \Pi} \gamma_j \right), \quad i = 1, 2, ..., m, \\
\sum_{i=1}^{m} \sum_{j \in \Pi} \alpha_{ij} \xi_{ij} \geq \sum_{j \in \Pi} \gamma_j, \quad r = 1, 2, ..., z, \\
0 \leq \sum_{i=1}^{m} \alpha_{ij} \xi_{ij} \leq \sum_{j \in \Pi} \gamma_j, \quad i = 1, 2, ..., m, \\
\lambda \in \Omega.
\]

It is easily seen that:

\[
\lambda = (\hat{\lambda}_j : j \in \Pi) \in \Omega.
\]

By Eq 13 and $\hat{\theta}_M < 0$, we have:

\[
\sum_{j \in \Pi} \hat{\lambda}_j x_j \leq \hat{\theta}_m x_M = \hat{\theta}_m \sum_{j \in \Pi} \alpha_{ij}^*.
\]

Without loss of generality, we assume that $\alpha_i k^* > 0$, because $x_M \neq 0$. By Eq 16, we get

\[
\sum_{j \in \Pi} \hat{\lambda}_j x_j \leq \hat{\theta}_m \sum_{j \in \Pi} \alpha_{ij}^*.
\]

Therefore, there exists a positive scalar $\mu > 0$, such that

\[
\sum_{j \in \Pi} \hat{\lambda}_j x_j \leq \hat{\theta}_m \sum_{j \in \Pi} \alpha_{ij}^* + (\alpha_{ij}^* - \mu),
\]

and $\alpha_i k^* \mu > 0$.

Now, define

\[
\hat{\alpha}_{ij} = \begin{cases} \alpha_{ij}^* - \mu & \text{if } i = j = k, \\ \alpha_{ij}^* & \text{otherwise.} \end{cases}
\]
According to Eqs 13, 14, 15, and 17, it is obvious that is a feasible solution to model (6). The value of the objective function of LP (6) at this feasible point is equal:

\[\sum_{i=1}^{m} \alpha_i + \sum_{j \in \Pi} \omega_j (a_{ij} + a_{ik}) = \sum_{j \in \Pi} \omega_j (a_{ij} + a_{ik}) \leq \sum_{j \in \Pi} \alpha_j + \alpha_k \]

This contradicts the assumption and completes the proof.

Now consider in the following question:

**Question 2.** If DMU, \(j \in \Pi\) are required to merge and form a new unit (DMU, \(M\)), in which input vector and predetermined efficiency target of DMU, \(M\) are \(x_M = \sum_{j \in \Pi} x_j\) and \(\phi_M\), respectively, how much should be input vector of this new DMU?

Here, suppose DMU keeps the amount of inputs of all DMU, \(j \in \Pi\), and looking to find the maximum amount of outputs of these DMUs in order to reach the pre-defined target level. The aim of the Question 2 is estimating the output vector \(y_M\) provided that the efficiency index of DMU, \(M\) is \(\phi_M\). In other words, the optimal value of the following model is equal \(\phi_M\).

\[
\phi^* = \max \phi
\]

\[
s.t. \quad \sum_{j \in \Pi} \lambda_j x_g + \lambda_M x_M \leq x_M, \quad i = 1, 2, \ldots, m,
\]

\[
\sum_{j \in \Pi} \lambda_j y_g + \lambda_M y_M \geq \phi y_M, \quad r = 1, 2, \ldots, s,
\]

\[
\lambda \in \bar{\Omega},
\]

where \(x_M = \sum_{j \in \Pi} x_j\), \(i = 1, 2, \ldots, m\) and \(\bar{\Omega} = \left\{ \lambda = (\lambda_j; j \in \Pi, \lambda_M) \in \mathbb{R}^{m+1} \big| \begin{array}{c} s.t. \quad a_i \left( \sum_{j \in \Pi} \lambda_j + \lambda_M a_k + \lambda_M a_j \right) = \sigma_i, \sigma_i \geq 0 \end{array} \right\}\).

To estimate output vector of DMU, the following multiple objective non-linear programming (MONLP) problem is considered:

\[
\begin{aligned}
\max \quad & \sum_{r=1}^{m} \beta_{g+r} y_r \\
\text{s.t.} \quad & \sum_{j \in \Pi} \lambda_j x_g + \lambda_M x_M \leq \sum_{j \in \Pi} x_j, \quad i = 1, 2, \ldots, m,
\end{aligned}
\]

where \(\phi_M\) is a predetermined target for efficiency of the merged DMU, \(M\). In the above model, \(\beta_{g+r} \in \mathbb{R}^s, \lambda_M \in \mathbb{R}^m \) \(r = 1, 2, \ldots, s\), \(\forall j \in \Pi\) is the variables vector. Let us assume that \(\omega_r, r = 1, 2, \ldots, s\), are important degree of for each of outputs of the merged DMU, \(M\). Therefore, model (20) converted to the following only objective model:

\[
\begin{aligned}
\max \quad & \sum_{r=1}^{m} \omega_r y_r \\
\text{s.t.} \quad & \sum_{j \in \Pi} \lambda_j x_g + \lambda_M x_M \leq x_M, \quad i = 1, 2, \ldots, m,
\end{aligned}
\]

Similar to the discussion raised in the conversion of model (5) to model (6), the nonlinear model (21) can be transformed into the following linear model:

\[
\begin{aligned}
\max \quad & \sum_{r=1}^{m} \omega_r y_r \\
\text{s.t.} \quad & \sum_{j \in \Pi} \lambda_j x_g + \lambda_M x_M \leq \sum_{j \in \Pi} x_j, \quad i = 1, 2, \ldots, m,
\end{aligned}
\]

where \(\bar{\Omega} = \left\{ \lambda = (\lambda_j; j \in \Pi, \lambda_M) \in \mathbb{R}^{m+1} \big| \begin{array}{c} s.t. \quad \sum_{j \in \Pi} \beta_{g+r} \lambda_j = \sigma_{g+r}, \sigma_i \geq 0 \end{array} \right\}\). Where \(\lambda_M\) is a new variable in Model (22), the number of variables is strongly reduced, and so computational complexity is reduced.

The proof of the following theorem is omitted because it is similar to the proof of Theorem 2 in (Gattoufi et al., 2014).

**Theorem 2.3** Let \((\phi^*, \lambda_j^*; \forall j \in \Pi, \lambda_M^* \geq 0)\) be an optimal solution the following model:
where,

\[
x_{in+1} = \sum_{j \in \Pi} x_{ij}; (i=1, 2, \ldots, m)
\]

and

\[
y_{rn+1} = \sum_{j \in \Pi} y_{rj}; (r=1, 2, \ldots, s)
\]

Then, Model (22) is feasible if and only if \(\phi^* \geq \bar{\phi}_M\).

With the minor changes in the proof of Theorem 1.3, the following theorem can be proved. Therefore, the proof of Theorem 3.3 is omitted. Theorem 3.3 shows how Model (22) can be used to estimate of outputs of DMU_M.

**Theorem 3.3** Let DMU_j, \(\forall j \in \Pi\) be inefficient.

Then, Model (22) is feasible if and only if \(\phi^* \geq \bar{\phi}_M\).

With the minor changes in the proof of Theorem 1.3, the following theorem can be proved. Therefore, the proof of the theorem 3.3 is omitted. Theorem 3.3 shows how Model (22) can be used to estimate of outputs of DMU_M.

As can be seen, DMUs_5 and DMUs_14 are inefficient DMUs.

Suppose that the decision maker wants to establish a new DMU (DMU_M) by merging these DMUs, in which output vector and predetermined efficiency target of DMUs_M are \((y_1M, y_2M) = (24.65768, 27.48995)\) and \(\bar{\theta}_M = 0.9215\), respectively. To determine input vector DMU_M, Model (7) corresponding to DMU_M is written as follows:

\[
\begin{align*}
\min & \quad \alpha_1 x_{1M} + \alpha_2 x_{2M} \\
\text{s.t.} & \quad \sum_{j \in \Pi} \lambda_j x_{ij} \leq \bar{\theta}_M \alpha_1 \lambda_j, \\
& \quad \sum_{j \in \Pi} \lambda_j y_{rj} \leq \bar{\theta}_M \alpha_2 \lambda_j,
\end{align*}
\]

**AN EXAMPLE WITH REAL DATA**

Consider a static technology comprising of 14 the educational departments in Islamic Azad University of Khomeinishahr-Iran as DMU, in which each DMU to produce two different continuous-valued outputs, Satisfaction of the students (y_1) and Satisfaction of the professors and staff (y_2), uses two different continuous-valued inputs, Facilities (x_1) and Amount of the attention paid to the department by the university (x_2). The data is obtained from the work of Ghoudbi & Jahangiri (2015). The results of inputs, outputs and efficiency score (considering input-oriented BCC model) are shown in Table 1:

**Table 1**: The data and efficiency score under BCC.

<table>
<thead>
<tr>
<th>Departments</th>
<th>x_1</th>
<th>x_2</th>
<th>y_1</th>
<th>y_2</th>
<th>Efficiency Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU1</td>
<td>0.385854</td>
<td>0.782695</td>
<td>11.76842</td>
<td>13.97176</td>
<td>0.9772</td>
</tr>
<tr>
<td>DMU2</td>
<td>0.53634</td>
<td>0.786386</td>
<td>12.24444</td>
<td>10.01111</td>
<td>0.9520</td>
</tr>
<tr>
<td>DMU3</td>
<td>0.972344</td>
<td>0.852564</td>
<td>12.43333</td>
<td>15.98</td>
<td>1.0000</td>
</tr>
<tr>
<td>DMU4</td>
<td>0.554214</td>
<td>0.712929</td>
<td>11.27391</td>
<td>14.41182</td>
<td>1.0000</td>
</tr>
<tr>
<td>DMU5</td>
<td>0.358756</td>
<td>0.912581</td>
<td>12.50481</td>
<td>13.07813</td>
<td>0.8889</td>
</tr>
<tr>
<td>DMU6</td>
<td>0.417995</td>
<td>0.672647</td>
<td>9.646154</td>
<td>14.51444</td>
<td>1.0000</td>
</tr>
<tr>
<td>DMU7</td>
<td>0.511568</td>
<td>0.784326</td>
<td>12.31864</td>
<td>14.41182</td>
<td>1.0000</td>
</tr>
<tr>
<td>DMU8</td>
<td>0.388259</td>
<td>0.837351</td>
<td>13.24667</td>
<td>10.3875</td>
<td>1.0000</td>
</tr>
<tr>
<td>DMU9</td>
<td>0.558262</td>
<td>0.829015</td>
<td>12.28824</td>
<td>12.72222</td>
<td>0.9065</td>
</tr>
<tr>
<td>DMU10</td>
<td>0.272026</td>
<td>0.814224</td>
<td>12.34615</td>
<td>14.11111</td>
<td>1.0000</td>
</tr>
<tr>
<td>DMU11</td>
<td>0.198246</td>
<td>0.883972</td>
<td>11.55625</td>
<td>12.77</td>
<td>1.0000</td>
</tr>
<tr>
<td>DMU12</td>
<td>0.546817</td>
<td>0.748349</td>
<td>12.48148</td>
<td>14.41182</td>
<td>1.0000</td>
</tr>
<tr>
<td>DMU13</td>
<td>0.558458</td>
<td>0.952591</td>
<td>13.03182</td>
<td>14.41182</td>
<td>1.0000</td>
</tr>
<tr>
<td>DMU14</td>
<td>1</td>
<td>1</td>
<td>12.15287</td>
<td>14.41182</td>
<td>0.7387</td>
</tr>
</tbody>
</table>

As can be seen, DMUs_5 and DMUs_14 are inefficient DMUs.

\[\text{If } \Lambda = (\lambda_j : j \in \Pi, \beta_j : r = 1, 2, \ldots, s, \forall j \in J) \text{ is an optimal solution with optimal value of } \sum_{r=1}^{s} \sum_{j \in \Pi} \omega_i \beta_{rj} \text{ to model (22), then efficiency score of DMU}_M \text{ with input vector } x_M = \sum_{j \in \Pi} x_{ij} \text{ and output vector } y_M = \sum_{j \in \Pi} y_{rj} \text{ is } \bar{\phi}_M.\]
Considering different important degree $\omega_1$ and $\omega_2$, in which $\omega_1 + \omega_2 = 1$ for each of inputs of $DMU_M$, the following solutions are generated:

$(x_{M_1}, x_{M_2}) = (\alpha_{M_1}^*, \alpha_{M_2}^*) = (1.17, 1.91),
(x_{M_1}, x_{M_2}) = (\alpha_{M_1}^*, \alpha_{M_2}^*) = (1.26, 1.91)$.

Therefore, if the educational departments of $DMU_5$ and $DMU_{14}$ are required to merge and form a new DMU with predetermined efficiency target of and output vector then $DMU_M$ must receive inputs such as one of the above solutions.

**CONCLUSION**

In the present paper, a novel method proposed to estimate inputs/outputs in the problem of merging DMUs in order to reach a predetermined efficiency target. Sufficient conditions are introduced to find the minimum/maximum amount of inputs/outputs of merging DMUs in order to reach the pre-defined target level. Our method, unlike other proposed method (Gattoufi et al., 2014), decreases the number of the variables of the model strongly, and this decreases the computational complexity. Also, a numerical example with real data is presented to confirm the credibility and applicability of our method.

Here, following research topics are recommended:

- Obtaining necessary conditions to estimate inputs/outputs.
- Similar models can be investigated for merging efficient DMUs.
- Similar models can be developed in presence of fuzzy data.
- Similar models can be developed in presence of stochastic data.

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