Iranian Journal of Optimization
Volume 10, Issue 1, 2018, 1-7
Research Paper
Online version is available on: www.ijo.iaurasht.ac.ir

Islamic Azad University
Rasht Branch
E-ISSN:2008-5427

A Stochastic Model for Water Resources Management

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Received: 17 June 2017
Accepted: 07 August 2017

Keywords:
water management
Two-stage stochastic programming
probabilistic programming
stochastic modeling

Abstract

Irrigation water management is crucial for agricultural production and livelihood security in many regions and countries throughout the world. Over the past decades, controversial and conflictladen water-allocation issues among competing municipal, industrial and agricultural interests have raised increasing concerns. Particularly, growing population, varying natural conditions and shrinking water availabilities have exacerbated such competitions. Shrinking water availabilities can result in reduced water supplies, while growing population can lead to increased water demands, these two facts can further intensify the water shortage. Stochastic programming methodology is applied in this paper to a capital investment problem in water resources. A framework is offered for the evaluation of electricity generation and water supply for agricultural irrigation. This essessment is conducted through the construction of an appropriate stochastic optimization model. A recursive least squares algorithm is incorp-orated in the model which enablee more accurate estimation of model parameters.

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INTRODUCTION

Stochastic programming deals with situations where some random parameters appear in a formulation of a mathematical program. Decision models of stochastic programming have been designed to treat the cases when a decision has to be chosen before a realization of random parameters can be observed. Examples of stochastic programming are athematical programs resulting from such problems as optimizing the stochatic inflows of a reservoir power system (Trezos & Yeh, 1987) using probabilistic models in the design and operation of a reservoir system (Arunkumar & Yeh, 1973) reservoir management and operations models (Yeh, 1985) and others. This paper addresses an application of stochastic programming to a water resource system with a dual-purpose of generating electricity and supplying water for agricultural irrigation. This paper aims to formulate the dual-purpose water resource system ss a change-constrained explore the use of recursive least squares filtering (Geib, 1974) in the model, and examine and discuss properties of the model.

With limited water resources for agriculture irrigation, managers tried to identify desired alternatives through raising irrigation productivity to realize optimal water allocation (Ma et al., 2014; Li et al., 2015; Wang et al., 2015) Two-stage stochastic programming (TSP) is effective to deal with problems for which an analysis of policy scenarios is desired and the uncertainties can be expressed as probabilistic distributions. In TSP, an initial decision (first-stage decision) is made based on uncertain future events, then an action can be taken after the preregulated disclosure of random variables (second-stage decision) (Li et al., 2006). This implies that TSP can minimize the expected costs of all applicable decisions taken over the two periods. TSP has been widely applied to water resources management over the past few decades (Maqsood et al.,2005; Wang & Huang, 2011).

Generally, in the design and operation of any water reservoir system, the stochastic nature of inflows must be taken into consideration. A multitude of techniques have been employed to model the stochastic inflows to water reservoir systems. In the literature, most researchers have applied stochastic dynamic programming (SDP) for single or multi-reservoir operation optimization. This is because SDP permits integration of stochastic elements in the model and the sequential nature of the problem. Each optimization technique has its own restriction on the type of constraints it can handle. For instance, a linear program entails all constraints in a linear form, whereas a dynamic program entails that they be expressed as functions of the present, and be independent of the previous variables (see, for example, (Butcher, 1971; Stedinger et al., 1987). In the study by Stedinger et al. (1984), SDP wss used to calculate optimal single reservoir operation.

In this instance, SDP can derive reservoir operating policies which are subject to reliability constraints. A different approach to modeling the system has been applied by Askew (1974), who formulates the problem as a chance-constrained dynamic program (CCDP), an SDP which has the resources to confine the probability of given variables taking on values outside a fixed range. Askew (1974) has also considered the trade-off of benefits and risks associated with long-term reservoir operations. He discusses reliability-constrained dynamic programm-ing that provides control over the probability of failure of the modeled reservoir system. His formulations are based on dynamic programming with discounting. Butcher discusses a multipurpose single reservoir system extensively. He has devised a deterministic optimal operating policy based on dynamic programming. In his models, sets of deterministic streamflows recur in a Monte Carlo study. Butcher employs a backward dynamic recursive equation using Bellman's Principle of Optimality. The following general forulation is developed by Butcher:

$$f_i(s_i,q_{i+1}) = \max_d \left[R(d) + \sum_{q_i=0}^{q_i max} P(q_i|q_{i+1}). f_{i+1}(s_i + q_i - d - e_i,q_i)\right]$$

for all values of sir the storage at the start of the second time period and all possible values of inflow during the preceding period q_(i+1). There exists a value of the released d which makes the right-hand side of the model 1 a maximum. This general formulation uses the fact that the inflow in any month is connected to the flow in the preceding month by the conditional probability, $P(q_i |q_{(i+1)})$.R(d) is the return obtained by releasing a quantity of water d in the ith time period, and e_i is the loss of water in storage by evaporation. Therefore, the general formulation gives $f_i (s_i ,q_{(i+1)})$, which is the expected return from the optimal operation of a system which has time periods to the end of the planning period.

Taylor and Karlin (1984) employ a Markov decision model to a water resource system by specifying the law of motion, *q(s' , q_{i+1})*. This is a function given when the probability of a particular state s^' being chosen as a joint result of the current state s and the chosen action a. In their study, they have defined S_n, to be the level of water at period n, A_n the amount released during the period, and I_n the input to the storage system during period n and based on the probability distribution, $Pr\ (I_n=k)=p(k)$ *fork=0,1,...* . Assumptions are made so that a reservoir has a maximum capacity of M and that This will lead to the transition law,

$$q(s'|s,a)=Pr\{I_n=s'-s+a\}=p(s'-s+a, \qquad if\ 0\leq s'\leq M$$

and

$$q(M|s,a)=Pr\{I_n\geq M-s+a\}=\sum_{s^\wedge\geq M} p(s'-s+a).$$

We shall analyze a water resource system similar to that of Taylor and Karlin (1984). Nonetheless, a different approach will be taken to formulate the problem.

TWO-STAGE STOCHASTIC PROGRAMMING

Two-stage stochastic programming (TSP) refers to a tradeoff between predefined strategies and the associated adaptive adjustments. In TSP, an initial first-stage decision must be made before the random variable is observed, then an action can be taken after the pre-regulated disclosure of second-stage decision . TSP cannot only handle uncertainties expressed as random variables but also provide an effective linkage between the pre-regulated policies and the associated economic implications caused by improper policies. Generally, a TSP model can be formulated as follows:

$$\begin{aligned} \min f &= cx - E[Q(x,\xi)] \\ s.t. \quad Ax &\leq b, \qquad x \geq 0 \end{aligned} \tag{1}$$

Where x is the first-stage decision made before the random variable is observed, ξ is the random variable ($\xi\in\Omega$), and $Q(x,\xi)$ is the optimal value of the following nonlinear programming:

$$\begin{aligned} &\min q(y,\xi) \\ s.t.\ &W(\xi)y = h(\xi) - T(\xi)x \qquad x\geq 0 \end{aligned} \tag{2}$$

where y is the second-stage adaptive decision, which depends on the realization of the random variable. $q(y,\ \xi)$ denotes the secondstage cost function, while $\{T(\xi),\ W(\xi),\ h(\xi)|\in\Omega\}$ are random model parameters with reasonable dimen-sions, which are functions of the random variable ξ. The first-stage decision is made before the random variable is observed. Then, when the random variable is observed, the discrepancy that may exist between $h(\xi)$ and $T(\xi)x$ corrected by recourse action that minimizes $q(y,\ \xi)$ and satisfies $W(\xi)y=h(\xi)-T(\xi)x,\ y\geq 0$. The pre-regulated cost and the potential penalty can thus be taken into account. Therefore, model 1 can be reformulated as follows:

$$\begin{aligned} &\max f = cx - E\left[\min_{y\geq 0}\{q(y,\xi)|T(\xi)x + W(\xi)y = h(\xi)\}\right] \\ &s.t: \qquad Ax \leq b, \\ &\qquad\qquad x\geq 0. \end{aligned} \tag{3}$$

Let the random variable ξ take discrete values ξ_1 with a probability level p_1 , where *l=1, 2,..., n.* It is assumed that $p_l>0$ and $\sum_{l=1}^{n} p_1=1$ The expected value of the second-stage optimization problem can be expressed as:

$$EQ(x) = E[Q(x,w)] = \sum_{i=1}^{n} p_l Q(x,\xi_l) \tag{4}$$

For each realization of random variable ξ_l, a second-stage decision is made, which is denoted by ξ_l. The second-stage optimization problem can then be written as:

$$\begin{aligned} &\min q(y,\xi) \\ s.t.\ &W(\xi_l)y = h(\xi_l) - T(\xi_l)x,\ \ \forall l = 1,2,\ldots,n \\ &y_l \geq 0 \end{aligned} \tag{5}$$

Through combining models 4 and 5, model 3 can be reformulated as follows:

$$\begin{aligned} &max f = cx - \textstyle\sum_{l=1}^{n} p_l\, q(y_l,\xi_l) \\ &s.t. \qquad Ax \leq b, \\ &T(\xi_l)x + W(\xi_l)y = h(\xi_l),\ \ \forall l = 1,2,\ldots,n \\ &y_l \geq 0 \end{aligned} \tag{6}$$

LINEAR PROGRAMMING MODEL TO MAXIMIZE TOTAL PROFIT

This stochastic programming formulation considers probability conditions on constraints. This formulation was first proposed by Revelle et al. (1969) for optimization of a reservoir system. We should use this formulation for a water reservoir system with a dual-purpose of generating electricity and supplying water for agricultural irrigation. The following are definitions of parameters in the system:

L_n:the water level (quantity) at the start of the period n","

Q_n: the amount of water released during period n"," i.e., the outflow of water, and

I_n: rainfall to the water reservoir system during period n"," i.e., the inflow.

It is assumed that I_n is a random variable independent from period to period. In addition, it will be treated as if it is deterministic. To achieve an equilibrium in the system, the following
equation must affirm:

New Level= Old Level + Inflow – Outflow. (7)

Mathematically

$$L_{n+1}=L_n+I_n-Q_n \quad (8)$$

or

$$L_{n+1}-L_n-Q_n=I_n \quad (9)$$

Assuming that the reservoir has a maximum cpacity it follows that

$$L_n \leq M, \ for\ all\ \ n=1, 2,\ldots, N \quad (10)$$

and thus,

$$L_{n+1}=min\{M, L_n+I_n-Q_n\} \quad (11)$$

It is also assumed that there exists a target water level of e units per period for electrical generation, and that there is no income value for exceeding the target level. Let E be the unit income for electrical use and K be the unit penalty for electrical shortages. Hence, the profit from electrical generation, $E.\ P_{(n+1)}$, in period $n+1$, is

$$E.P_{n+1}=E.min\{L_{n+1}, e\}-K.max\{0, e-L_{n+1}\} \quad (12)$$

It is assumed that water released for irrigation has a unit value of W. The profit from irrigation in period $n+1$, $IP_{(n+1)}$ is dependent on the outflow of water during period n. This relationship is depicted as follows:

$$IP_{n+1}=WQ_n \quad (13)$$

Therefore, the expected total profit in period $n+1$, TP_{n+1} is sum of model 12 and 13 Mathematically, it is written as follows:

$$TP_{n+1}=E.P_{n+1}+IP_{n+1}=E.min\{L_{n+1},e\}-K.max\{0,e-L_{n+1}\}+W.O_n \quad (14)$$

We can then formalize the optimization problem given the objective function in model 14 and constraints in 9 and 10. Therefore, the formulation of water reservoir program is

$$\begin{aligned} &max\, TP_{n+1}=E.P_{n+1}+IP_{n+1}=E.min\{L_{n+1},e\}-K.max\{0,e-L_{n+1}\}+W.O_n \\ &s.t. \quad L_{n+1}-L_n+Q_n=I_n, \\ &\quad\quad L_n \leq M, \\ &\quad\quad Q_n, I_n, L_n, L_{n+1} \geq 0, \quad n=1,\ldots,N \end{aligned} \quad (15)$$

However, the objective function in model 2 is not linear. Hence, to 15 , we shall define

$$W_{n+1}=min\{L_{n+1},e\}, \quad (16)$$

so that

$$W_{n+1} \leq L_{n+1} \quad (17)$$

and

$$W_{n+1} \leq e, \quad (18)$$

Also, by defining

$$X_{n+1}=max\{0, e-L_{n+1}\}, \quad (19)$$

It follows that

$$X_{n+1} \geq 0, \quad (20)$$

and

$$X_{n+1} \geq e-L_{n+1}, \quad (21)$$

Holds. Given the new characterizations, 15 become a simple LP model. Consequently,

$$\begin{aligned} max \quad & TP_{n+1} = E.W_{n+1} - K.X_{n+1} + W.O_n \\ s.t. \quad & L_{n+1} - L_n + O_n = I_n, \\ & X_{n+1} + L_{n+1} \geq e, \\ & W_{n+1} \leq e, \\ & L_n \leq M, \\ & O_n, W_{n+1}, L_n, L_{n+1}, X_{n+1}, \quad n = 1, \ldots, N. \end{aligned} \quad (22)$$

UNCERTAINTY ABOUT PARAMETERS OF A THEORETICAL DISTRIBUTION

In our previous discussion, we have treated I_n as deterministic. However, in this case, we are considering rainfall I_n to be a random variable with an arbitrary distribution with mean μ and variance σ^2 and that the observations are independent. Therefore, summing the following constraint

$$L_{n+1} - L_n + O_n = I_n, \quad (23)$$

from 1 to some will result in the following equation

$$\sum_{n=1}^{k}(L_{n+1} - L_n + O_n) = \sum_{n=1}^{k} I_n \quad (24)$$

$$L_{k+1} - L_1 + \sum_{n=1}^{k}(O_n) = \sum_{n=1}^{k} I_n \quad (25)$$

For a sufficiently large k, $\sum_{n=1}^{k} I_n$ is normally distributed by the Central Limit Theorem. Dividing both sides of model 25 by k, we shall obtain the following result:

$$\frac{L_{k+1} - L_1}{k} + \frac{1}{k}.\sum_{n=1}^{k} O_n = \frac{1}{k}.\sum_{n=1}^{k} I_n \quad (26)$$

We can further simplify model 26 to

$$\frac{L_{k+1} - L_1}{k} + \bar{O}_k = \bar{I}_k \quad (27)$$

where

$$\bar{O} = \frac{1}{k}.\sum_{n=1}^{k} O_n \quad (28)$$

$$\bar{I} = \frac{1}{k}.\sum_{n=1}^{k} I_n \quad (29)$$

Given these criteria, kindependent and identically distributed random variables, 22 can be represented as a linear system with linear constraints, the presumption of maximum admissible probabilities of inflows can be managed by chance-constrained linear rogramming (CLP) techniques. These techniques were first consummated by Charnes and Cooper (1963) and have been illustrated by Revelle et al. (1969):

$$\begin{aligned} & maxZ = c^T x, \\ & s.t. \quad Ax = b, \\ & P[Tx \geq p] \geq \alpha, \\ & x \geq 0 \end{aligned} \quad (CLP)$$

Where $P[0\ ]$ denotes probability, α is a constant vector, c is cost coefficient vector, b is a right-hand side vector, A and T are coefficient matrices, and lastly, x is a decision vector.

THE PROBABILISTIC MODEL

For a dual-purpose water reservoir system, a chance-constrained stochastic program is appropriate and is formulated as follows:

$$\begin{aligned} max \quad & TP_{k+1} = E.W_{k+1} - K.X_{k+1} + W.O_k, \\ s.t. \quad & P\{L_{k+1} + O_k \leq \textstyle\sum_{n=1}^{k} I_n - \gamma_{k-1}\} \geq 1 - \alpha, \\ & X_{k+1} + L_{k+1} \geq e, \\ & W_{k+1} \leq e, \\ & W_{k+1} - L_{k+1} \leq 0, \\ & L_{k+1} \leq M, \\ & L_{k+1}, X_{k+1}, O_k \geq 0, \quad k = 1,2, \ldots \end{aligned} \quad (PWRP)$$

where γ_{k-1} is equal to $\sum_{(n=1)}^{(k-1)} O_n - L_1$ and is constant because sample data are known. 1-α is the level of significance.

The above probabilistic water reservoir program (PWRP) is equivalent to a deterministic model which is a linear program.

THE DETERMINISTIC MODEL

Our procedure in using rainfall data samples to estimate the parameter p will follow the recursive least-squares approach. This algorithm will enable us to recursively revise the estimate at time k to reflect the most recent data sample obtained in period k+1. The parameter cr, on the other hand, is estimated from the maximum likelihood formula for nonoverlapping blocks of data samples as shown below. Given most recent estimates of the parameters, we then use this information to formulate a deterministic model.

Since there may be zero rainfalls for several periods, the variance estimate should be based on a sufficiently large block of data. To compute the variance of the rainfall variable for every block of some k data points, we use the maximum likelihood estimation formula

$$\sigma_k^2 = \frac{1}{k-1}\sum_{n=1}^{k}(I_n - \bar{I}_k)^2$$
$$\sigma_{2k}^2 = \frac{1}{k-1}\sum_{n=k+1}^{2k}(I_n - \bar{I}_{2k})^2, \dots, \quad (30)$$

where

$$\bar{I}_{2k} = \frac{1}{k}\sum_{n=k+1}^{2k} I_n. \quad (31)$$

Assume that we have a historical (a priori) estimate of the variance, and that the updating will be done at the sample periods t_k , k=1, 2, 3 and so on. A simple method for updating the sample variance for period t from the last available value $\sigma_{(t-1)}^2$ is through the exponential smoothing formula

$$\sigma_1^2 = \sigma_0^2 + \lambda(\sigma_k^2 - \sigma_0^2),$$
$$\sigma_2^2 = \sigma_1^2 + \lambda(\sigma_{2k}^2 - \sigma_1^2),$$
$$\vdots \quad (32)$$
$$\sigma_t^2 = \sigma_{t-1}^2 + \lambda(\sigma_{tk}^2 - \sigma_{t-1}^2), \dots ,$$

where is the smoothing constant which assumes a value in the range [0,1].

The recursive least-squares method for updating the mean parameter μof the rainfall distribution does not require normality in the data. The properties of the updated estimates $\hat{\mu}_n$ (+) are unbiasedness and minimum variance. The plus symbol in $\hat{\mu}_n$ (+) denotes that it is a posterior estimate. Thus the prior estimate, before the measurement I_n is received, is denoted by $\hat{\mu}_n$ (-).

The recursive updating formula for the mean is given as follows:

$$\hat{\mu}_n(+) = \hat{\mu}_n(-) + \frac{S_n^2}{\sigma_t^2}\{I_n - \hat{\mu}_n(-)\}, \quad (33)$$

where σ_t^2 is the most recent estimate of the variance by the block approach in Eq. 32 and S_n^2 (+)=E [$\hat{\mu}_n$ (+)-μ] ^{2} posterior error variance of the mean estimate S_n^2 (+) can be recursively computed from the prior, S_n^2 (-)=E [$\hat{\mu}_n$ (-)-μ]^2 by the recursive equation

$$\frac{1}{S_n^2(+)} = \frac{1}{S_n^2(-)} + \frac{1}{\sigma_t^2}, \quad n = 1,2,\dots,k,\dots,2k,\dots \quad (34)$$

and $t=0$ for $n<k$, $t=1$ for $k<n<2k, \dots$.

To start the recursive algorithm, we need some unbiased estimates S_n^2 (-), $\hat{\mu}_1$ (-) and σ_0^2. We will employ the posterior estimate, $\hat{\mu}_n$ (+), and σ_t^2 in the deterministic model for sequential planning of the dual-purpose water resource system.

We can now proceed to formulate a deterministic model in terms of the posterior estimates from the above algorithm:

$$\begin{aligned}
&\max \quad TP_{n+1} = E.W_{n+1} - K.X_{n+1} + W.O_n,\\
&s.t. \quad L_{n+1} + O_n \geq n\hat{\mu}_n(+) + z_{1+\alpha}\sqrt{n}\sigma_t - \gamma_{n-1},\\
&X_{n+1} + L_{n+1} \geq e, \qquad\qquad \text{(DWRP)}\\
&W_{n+1} \leq e,\\
&W_{n+1} - L_{n+1} \leq 0,\\
&L_{n+1} \leq M,\\
&L_{n+1}, X_{n+1}, O_n \geq 0, \quad n = 1,2,\dots,k,\dots,2k,\dots
\end{aligned}$$

$t=0$ for $n < k$, $t=1$ for $k < n < 2k$ and so on, and where $z_{(1-\alpha)}$ is the standard-ized 100(1-α)th normal quantile.

CONCLUSIONS

This paper provides an analytical framework for the evaluation of electricity generation and water supply for agricultural irrigation. The optimal total profit was determined by a chance-constrained linear program. We believe that through the incorporation of Recursive Least-Squares (RLS) estimates and the variance updating procedure in the model, it enables more accurate estimates of the uncertain rainfall process parameters. Further, the RLS procedure makes it easy to extend beyond the (assumed) independently, identically distributed rainfall measurements to random walk and Gauss-Markov processes. Then we could transform our CLP into a probabilistic model which is equivalent to a deterministic model. Another exten-sion is possible through using other formulations such as stochastic linear programming with recourse. In this program, the solution is obtained by making decisions in multiple stages. This modeling approach permits random variables to be incorporated in the constrained set of an LP problem. Hence, this approach is applicable to our water resource system problem. Future research is needed to test the applicability of our models under various conditions.

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