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Animal Diet Formulation with Floating Price

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Abstract

In the process of milk production, the highest cost relates to animal feed. Based on reports provided by the experts, around seventy percent of dairy livestock costs included feed costs. In order to minimize the total price of livestock feed, according to the limits of feed sources in each region or season, and also the transportation and maintenance costs and ultimately milk price reduction, optimization of the livestock nutrition program is an essential issue. Because of the uncertainty and lack of precision in the optimal food ration done with existing methods based on linear programming, there is a need to use appropriate methods to meet this purpose. Therefore, in this study formulation of completely mixed nutrient diets of dairy cows is done by using a fuzzy linear programming in early lactation. Application of fuzzy optimization method and floating price make it possible to formulate and change the completely mixed diets with adequate safety margins. Therefore, applications of fuzzy methods in feed rations of dairy cattle are recommended to optimize the diets. Obviously, it would be useful to design suitable software, which provides the possibility of using floating prices to set feed rations by the use of fuzzy optimization method.

Keywords:
Diet
Fuzzy linear programming
floating price
least-cost
Livestock

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INTRODUCTION

Livestock ration formulation models have been developed for commercial purposes as well as for its development, using various mathematical techniques for several decades. Some of them are Pearson square method, two by two matrix method, and trial and error method. Some mathematical programming techniques are being used to formulate the ration such as linear programming, multiple objective programming, goal programming, separable programming, quadratic programming, nonlinear programming, and genetic algorithm. Linear programming is the common method of least cost feed formulation and for the last fifty years, it has been used as an efficient technique in ration formulation (Gupta et al., 2013).

In milk production, the main costs are related to animal feed. Therefore, to reduce the total price of milk, the use of diets with the lowest price is essential. The linear programming model can be applied to find the lowest price that will meet all the needs of lactating cows. However, the parameters of this model are often considered as definitive, which is not real and are by approximate nature. Meeting the real needs of food rations during the use of these models is impossible. Animal feed price is usually stated as an interval or range, and are not accurate estimates, because it has always been volatile and may vary from region to region. Transport and harvest costs and above all the existence of feed material in a region may affect on total cost of feed. So fuzzy sets and numbers are useful tools for modelling of such uncertain and imprecise problems.

Fuzzy logic was first proposed by Zadeh in 1965. Fuzzy set theory has been applied to many disciplines such as control theory and management sciences, mathematical modelling and industrial applications. Concept of decision analysis in fuzzy environment was first proposed by Bellman and Zadeh. Fuzzy Linear Programming (FLP) is an especially useful and practical model for many real world problems(Bellman et al., 1970). Tanaka et al was first introduced the concept of fuzzy linear programming in general level(Tanka et al,1974). Zimmermann first used the max-min operator of Bellman and Zadeh to solve FLP problems. Zimmermann and Iskander has been used Max-min operator in solving other types of fuzzy programming (Zimmermann, 1987; 2001). Maleki used a certain ranking function to solve fuzzy linear programming problems(Maleki et al., 2000). Zhang et al. proposed a method for solving fuzzy linear programming problems which involve fuzzy numbers in coefficients of objective functions(Zhang et al., 2003). Ganesan and Veeramani introduced a new method for solving a kind of linear programming problems involving symmetric trapezoidal fuzzy numbers without converting them to crisp linear programming problems based on the primal simplex method(Ganesan et al., 2006). Mahdavi-Amiri and Nasseri have also explored some duality properties for two kinds of FLP problems, namely Fuzzy Number Linear Programming (FNLP) and Fuzzy Variable Linear Programming (FVLP) problems by using a linear ranking function(Mahdavi amiri et al., 2006؛ 2007). Nasseri proposed a method for solving fuzzy linear programming problems by solving the classical linear programming(Nasseri, 2008). Ebrahimnejad and Nasseri used the complementary slackness theorem to solve fuzzy linear programming problem with fuzzy parameters without the need of a simplex tableau(Ebrahimnejad et al., 2009). Ebrahimnejad et al. proposed a new primal-dual algorithm for solving linear programming problems with fuzzy variables by using duality results(Ebrahimnejad et al., 2010). Nasseri and Ebrahimnejad proposed a fuzzy primal simplex algorithm for solving the flexible linear programming problem(Nasseri et al., 2010b). Ebrahimnejad et al developed the bounded simplex method for solving a special kind of linear programming with fuzzy cost coefficients, in which the decision variables are restricted to lie within lower and upper bounds(Ebrahimnejad et al., 2011). Many authors considered various types of the FLP problems and proposed several approaches for solving these problems (Ebrahimnejad et al., 2009؛ Ganesan et al., 2006؛ Mahdavi amiriet al., 2009؛ Maleki et al., 2000؛ Nasseri, 2008؛ Nasseri et al., 2009).The diet problem was one of the first optimization problems studied in the 1930s and 1940s (Marinov et al., 2014). Linear programming techniques have been extensively used for animal diet formulation for more than last fifty years. To overcome the drawback of linear approximation of the objective function for diet formulation, a mathematical model based on nonlinear programming technique is proposed to measure animal performance in terms of milk yield and weight gain (Sexena, 2011). Some biological optimization problems imply finding the best compromise among several conflicting demands in a fuzzy situation. For example, experimental results show that a micro-organism may reflect the resilience phenomenon after stressful environmental changes and genetic modification (Wang et al., 2014).

Fuzzy linear programming is considered as an appropriate method for solving the problems of dairy cows diets when feeding prices used in completely mixed diets are expressed as fuzzy numbers. In this case, all the numerical coefficients that are expressed as approximate and imprecise can be stated in terms such as approximately, about or in range. On the other hand, in completely mixed diet formulation, if feeding prices are expressed in the interval, their meaning may not provide good information about that interval; while if the membership functions of these intervals were available, decisions would have been made on the basis of information that is more complete. In this context because of flexibility in choosing the coefficients, fuzzy linear programming can help more effectively to decide about the appropriate food formulation. Since it is helpful for better modelling of the inherent uncertainty that the user faced about the feed rations data base. Linear programming, fuzzy linear programming and mathematical techniques, techniques have been extensively used for diet formulation (Bas, 2014; Hsu et al., 2013; Mamt et al., 2012; Moraes et al., 2012 ؛Piyaratne et al., 2012). Cadenzas et al. presented the application of fuzzy optimization to diet problems in Argentinean farms and also developed software which has several capabilities such as the suggestion of different diets with satisfied diet requirements, price and the amount of constraint satisfaction (Cadenas et al., 2004). Castrodeza et al. gave a multi-criteria fractional model for feed formulation with economic, nutritional and environmental criteria. Together with the search for the lowest possible cost, they introduced some other aspects such as maximizing diet efficiency and minimizing any excess that may lead to unacceptable damage to the environment (Castrodes et al., 2005). Pomar et al developed multi-objective optimization model based on the traditional least-cost formulation program to reduce both feed cost and total phosphorus content in pig feeds (Pomar et al., 2007). Niemi et al. used stochastic dynamic programming to determine the value of precision feeding technologies for grow-finish swine (Niemi, 2010). Darvishi et al used fuzzy optimization in diet formulation and using a fuzzy model in comparison to linear programming models, feed costs was reduced to about 8 percentages. The result of this experiment guarantees the formulation of ration using fuzzy models can be used to reduce feed cost and obtain different ration that they may met dairy cow nutrient requirements over different situations (Ebrahimnejad et al., 2010). Mamat et al concentrates on the human diet problem using fuzzy linear programming approach. This research aims to suggest people have healthy food with the lowest cost as possible (Mamat et al., 2011).

In order to develop the decision making approach of Operations Research (OR) in the other subjects, Fuzzy and Stochastic approaches are used to describe and treat imprecise and uncertain elements present in a real decision problem. In fuzzy programming problems the constraints and goals are viewed as fuzzy sets and it is assumed that their membership functions are known. In this paper, linear programming with fuzzy cost coefficients was used for the completely mixed rations of lactating cows in early lactation intake (from birth to 70 days postpartum), weighing between 600 to 700 kg. In order to determine the fluctuation of food prices, fuzzy theory approach was employed, where the prices of food were assumed as fuzzy numbers.

The remainder of this paper is organized as follows. In Section 2, we first give some necessary notations and definitions of fuzzy set theory. Then we provide a discussion of fuzzy numbers and linear ranking functions for ordering them. In particular, a certain linear ranking function for ordering trapezoidal fuzzy numbers is emphasized. In Section 3, explains the notion of linear programming with fuzzy cost coefficients, basic feasible solution and fuzzy simplex algorithm for linear programming problems with fuzzy cost coefficients. To illustrate some materials and methods in diet formulation of livestock in Section 4 and using the linear programming with fuzzy cost coefficients model in a formulation of dairy cow ration in early lactation and solving it then analysed of the obtained results while Section 5 presents the conclusions.

PRELIMINARIES

Definitions and notations

In this section, we review some necessary backgrounds and notions of fuzzy set theory, initiated by Bellman and Zadeh (1970) and which are taken from (Mahdavi amiri et al., 2007; Maleki et al., 2000).

Definition 2.1. The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range, i.e. $\mu_A: X \rightarrow [0, 1]$ The assigned value indicates the membership grade of the element in the set A.

The function μ_A is called the membership function and the set $\tilde{A}=\{(x, \mu_{\tilde{A}}(x)), x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2.2. The height, $h(\tilde{A})$ of a set fuzzy $\tilde{A}$ is the largest grade obtained by any element in that set, i.e. $h(\tilde{A})=sup\{\mu\tilde{A}(x), x \in X\}$.

Definition 2.3. A fuzzy set $\tilde{A}$ is called normal if $h(\tilde{A})=1$.

Definition 2.4. A fuzzy set $\tilde{A}$ is called convex, if for each x, y$\in X$ and each $\lambda \in [0, 1]$, we have $\mu_{\tilde{A}}(\lambda x+(1-\lambda)y) \geq min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}$

Definition 2.5. A fuzzy number is a convex normalized fuzzy set of the real line ¬whose membership function is piecewise continuous.

Definition 2.6. A fuzzy number $\tilde{A}=(a^L, a^U, \alpha, \beta)$ is said to be a trapezoidal fuzzy number, if its membership function is given by the function:

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{x-(a^L-\alpha)}{\alpha} & a^L-\alpha \leq x \leq a^L \\ 1 & a^L \leq x \leq a^U \\ \frac{(a^U+\beta)-x}{\beta} & a^U \leq x \leq a^U+\beta \\ 0 & else. \end{cases}$$

Remark 2.1. In this paper, we denote the set of all fuzzy numbers by

Definition 2.7. For a given α∈ [0,1], the α-level set of a fuzzy set $\tilde{A}$ is defined as an ordinary set A_α of elements x such that the membership function value $\mu_{\tilde{A}}(x)$ of x exceeds α, i.e., $A_\alpha=\{x \,|\, \mu_{\tilde{A}}(x) \geq \alpha\}$.

Definition 2.8. A trapezoidal fuzzy number $\tilde{A}=(a^L, a^U, \alpha, \beta)$ is said to be non-negative trapezoidal fuzzy number if and only if $a^L \geq 0$.

We next define arithmetic on trapezoidal fuzzy numbers. Let $\tilde{a}=(a^L, a^U, \alpha, \beta)$ and $\tilde{b}=(b^L, b^U, \gamma, \theta)$ be two trapezoidal fuzzy numbers. Define,

$$x \geq 0, \quad x\tilde{a}=(xa^L, xa^U, x\alpha, x\beta) \; x \in R,$$
$$x < 0, \quad x\tilde{a}=(xa^U, xa^L, -x\beta, -x\alpha) \; x \in R,$$
$$\tilde{a}+\tilde{b}=(a^L+b^L, a^U+b^U, \alpha+\gamma, \beta+\theta),$$
$$\tilde{a}-\tilde{b}=(a^L-b^U, a^U-b^L, \alpha+\gamma, \beta+\theta).$$

Definition 2.9. For any trapezoidal fuzzy number $\tilde{A}$ we define $\tilde{A} \geq \tilde{O}$ if there exist $\varepsilon \geq 0$ and $\alpha, \beta \geq 0$ such that $\tilde{A}=(-\varepsilon, \varepsilon, \alpha, \beta)$ We also denote $(-\varepsilon, \varepsilon, \alpha, \beta)$ by $\tilde{O}$.

Ranking functions

On the other hand, ranking of fuzzy numbers is an important issue in the study of fuzzy set theory. Ranking procedures are also useful in various applications and one of them will be in the study of fuzzy linear programming problems. Many methods for solving fuzzy linear programming problems are based on comparison of fuzzy numbers and in particular using ranking functions (Mahdavi amiri et al., 2007; Maleki et al., 2000). An effective approach for ordering the elements of F(R) is to define a ranking function R: F®→R which maps each fuzzy number into the real line, where a natural order exists.

Thus, we define the order on as follows:

$\tilde{a} \succeq \tilde{b}$ if and only if $\Re(\tilde{a}) \geq \Re(\tilde{b})$

$\tilde{a} \succ \tilde{b}$ if and only if $\Re(\tilde{a}) > \Re(\tilde{b})$

$\tilde{a} \approx \tilde{b}$ if and only if $\Re(\tilde{a}) = \Re(\tilde{b})$

Where $\tilde{a}$ and $\tilde{b}$ are $F(R)$ Also, we write $\tilde{a} \preceq \tilde{b}$ if and only if $\tilde{b} \succeq \tilde{a}$

We restrict our attention to linear ranking functions, that is, a ranking function $\Re$ such that $\Re(k\tilde{a}+\tilde{b})=k\Re(\tilde{a})+\Re(\tilde{b})$

For any $\tilde{a}$ and $\tilde{b}$ belonging to F(R) and any $k \in R$. Throughout this paper, we use linear ranking functions.

Lemma 2.1. Let $\Re$ be any linear ranking function. Then,

(i) $\tilde{a} \geq \tilde{b}$ if and only if $\tilde{a} - \tilde{b} \geq 0$ if and only if $-\tilde{b} \geq \tilde{a}$.

(ii) if $\tilde{a} \geq \tilde{b}$ and $\tilde{c} \geq \tilde{d}$ then $\tilde{a} + \tilde{c} \geq \tilde{b} + \tilde{d}$.

We consider the linear ranking functions on F(R) as $\Re(\tilde{a}) = c_L a^L + c_U a^U + c_\alpha \alpha + c_\beta \beta$ Where $\tilde{a} = (a^L, a^U, \alpha, \beta)$ and c_L, c_U, c_α, c_β are constants, at least one of which is nonzero.

A special version of the above linear ranking function was first proposed by Yager [36] as follows:

$\Re(\tilde{a}) = \frac{1}{2}\int_0^1 (\inf \tilde{a}_\lambda + \sup \tilde{a}_\lambda) d\lambda$ Which reduces to

$\Re(\tilde{a}) = \frac{a^L + a^U}{2} + \frac{1}{4}(\beta - \alpha)$ Then, for trapezoidal numbers $\tilde{a} = (a^L, a^U, \alpha, \beta)$ and $\tilde{b} = (b^L, b^U, \gamma, \theta)$ we have $\tilde{a} \geq \tilde{b}$ if and only if $a^L + a^U + \frac{1}{2}(\beta - \alpha) \geq b^L + b^U + \frac{1}{2}(\theta - \gamma)$.

LINER ROGRAMMING WHIT FUZZY COST COEFFICIENT

A number of researchers have shown interest in the area of fuzzy linear programming problems. A special kind of FLP problem in which all decision parameters except of decision variables are represented by trapezoidal fuzzy numbers is called fuzzy numbers linear programming problem. In this paper, we focus on a type of FNLP problems in which only the cost coefficients are represented by trapezoidal fuzzy numbers. Consider the following fuzzy linear programming problem.

Definition 3.1. Linear Programming problem with Fuzzy Cost Coefficients (LPFCC) is defined as follows:

$$\begin{aligned} \min \quad & \tilde{z} \approx \tilde{c}x \\ s.t. \quad & Ax = b \\ & x \geq 0 \end{aligned} \qquad (3.1)$$

Where $\tilde{c}^T \in (F(R))^n, b \in R^m, A \in R^{m \times n}$ are given $X \in R^n$ and is to be determined the concepts and the definitions are given from (Mahdavi amiri et al., 2007; Maleki et al., 2000).

Definition 3.2. Any vector $X \in R^n$ which satisfies the constraints and nonnegative restrictions of (3.1) is said to be a feasible solution.

Definition 3.3. Let S be the set of all feasible solutions of (3.1). Any fuzzy vector $X^* \in S$ is said to be a fuzzy optimum solution to (3.1) if $\tilde{c}x^* \preceq \tilde{c}x$ for all $x \in S$ where $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, ..., \tilde{c}_n)$ and $\tilde{c}x = \tilde{c}_1 x_1 + \tilde{c}_2 x_2 + ... + \tilde{c}_n x_n$.

Remark 3.1. Consider the system of constraints (3.1) where A is a matrix of the order and rank (A) =m. Any (m×n) matrix B formed by m linearly independent columns of A is known as a basis for this system. The column vectors of A and the variables in the problem can be partitioned into the basic and the nonbasic part with respect to this basis B. Each column vector of A, which is in the basis B, is known as a basic column vector. All the remaining column vectors of A are called the nonbasic column vectors.

Remark 3.2. Let be the vector of the variables associated with the basic column vectors. The variables in are known as the basic variables with respect to basis B, and X_B is the basic vector. Also, let X_n and N be the vector and the matrix of the remaining variables and columns, which are called the nonbasic variables and nonbasic matrix, respectively. In this case, $x = (x_B, x^N) = (B^{-1}b, 0)$ is a basic solution too.

Definition 3.4. Suppose $\bar{x}$ is a basic feasible solution of fuzzy system $Ax = b, x \geq 0$. If the number of positive variables $\bar{x}$ is exactly m, then it is called a non-degenerate basic feasible solution, i.e. $\bar{x} = (\bar{x}_1, \bar{x}_2, ..., \bar{x}_m) \succ (0, 0, ..., 0)$ If the number of positive $\bar{x}$ is less than of m, then is called a degenerate basic feasible solution.

Suppose $\bar{x}$ is a basic feasible solution of (3.1). Let y_k and w be the solutions to $By_k = a_k$ and $wB = \tilde{c}_B$, respectively and define $z_j = \tilde{w}a_j$.

Definition 3.5. We say that the real number a corresponds to the fuzzy number with respect to a given linear ranking function $\Re$, if $a = \Re(\tilde{a})$.

The following theorem shows that any FNLP can be reduced to a linear programming problem (see Maleki, 2002; Maleki et al., 2000).

Theorem 3.1. Fuzzy linear programming problem (3.1) is equivalent to the following linear programming problem:

$$\begin{aligned} &\min\ z=cx \\ &s.t.\quad Ax=b \\ &\qquad\quad x \geq 0 \end{aligned} \qquad (3.2)$$

where $cj=\Re(cj),\ j=1,\ldots n$.

Proof. Using a linear ranking function, the proof is simple. The reader can find the proof in detail in (Maleki et al., 2000).

Remark 3.3. The above theorem shows that the sets of all feasible solutions of fuzzy number linear programming (FNLP) problem and linear programming (LP) problem are the same. Also if $\bar{x}$ is an optimal feasible solution for FNLP, then $\bar{x}$ is an optimal feasible solution for LP.

For an illustration of Theorem 3.1 consider an example used by Maleki (2000).

Theorem 3.2. (Mahdavi amiri et al., 2007) Suppose the linear programming problem of fuzzy number is non-degenerate. A basic feasible solution $x_N=0\ x_B=B^{-1}b$, for the problem (3.2) is optimal, if and only if $\forall j, 1 \leq j \leq n, \tilde{z}_j \preceq \tilde{c}_j$.

Algorithm 3.1. The Fuzzy Primal simplex algorithm for LPFCC problem

Suppose a basic feasible solution with basis B and the corresponding simplex tableau is at hand.

1. The basic feasible solution is given by $x_B=B^{-1}b$ and $x_N=0$ the fuzzy objective value is: $\tilde{z} \approx \tilde{c}_B x_B$.

2. Calculate $y_{0j} = \Re(\tilde{z}_j - \tilde{c}_j)$, $j=1,\ldots,n,\ j\neq B_i$, $i=1,\ldots,m$. °Let $y_{0k}= \max_{j=1,\ldots,n}\{y_{0j}\}$ If then stop; the current solution is optimal.

3. If $y_k=B^{-1}a_k \leq 0$ then stop; the problem is unbounded. Otherwise, determine a variable index r corresponding to a variable x_B leaving the basis as follows:

$$\frac{b_r}{y_{rk}} = \min_{1\leq i\leq m}\{\frac{b_i}{y_{ik}} | y_{ik} > 0\}$$

4. Update the tableau by Pivoting at y_{rk} and update the simplex tableau with Gauss elimination operation (x_k enters the basic and x_{Br} leaves the basic). Go to Step (2).

The above algorithm is given from (Mahdavi amiri et al., 2009).

MATERIALS AND METHODS

The objective of the diet problem is to select a family of foods that will satisfy a set of daily nutritional requirement at a minimum cost. The classical diet problem is stated as a linear optimization problem (Marinov et al., 2014). Fuzzy Diet problems (FD) with a fuzzy objective in which, for each food $j=1,\ldots,n,$ there is some vagueness on its corresponding cost which is modelled by means of a fuzzy number defined by a membership function $\mu_j \in F(R)$ such that $\mu_j: R\rightarrow[0, 1]$ The problem can be formulated as follows:

$$\min \sum_{j=1}^{n} \tilde{c}_j x_j$$

$$s.t.\ \underline{b}_i \leq \sum_{j=1}^{n} a_{ij} x_j \leq \bar{b}_i, i=1,\ldots,m.$$

$$\underline{p}_j \leq x_j \leq \bar{p}_j,\ \ j=1,\ldots,n.$$

with regard to address a mathematical version of this problem, we need to define some variables. Let consider a set of foods $f=\{X_1, X_2,\ldots, X_n\}$ a set of nutrients $N=\{A_1, A_2,\ldots, A_n\}$ and the following variables:

$\tilde{c}_j$ = cost of the food $j, j = 1, \ldots, n,$

x_j = amount of food j to eat, $j = 1,\ldots, n,$

a_{ij} = amount of nutrient i in food $j,\ j=1,\ldots, n,\ i=1,\ldots, m,$

$\underline{b}_i$ = minimum amount of nutrient i required, $i=1,\ldots, m,$

$\bar{b}_i$ = maximum amount of nutrient i allowed per day, $i = 1,\ldots, m,$

$\underline{p}_j$ = minimum amount of food j desired per day, $j = 1,\ldots, n,$

$\bar{p}_j$ = maximum amount of food j desired per day, $j = 1,\ldots, n.$

Among several different food items used in livestock feed, only 11 foods (hay, grain, Sugar beet dried pulp, corn silage with dry matter between 32 and 38%, cottonseed meal with 41% CP, Calcium soaps, Sugar beet molasses, soybean meal with 44% CP, sunflower meal, wheat bran and Shell powder) used and for nutrients needs supply only seven nutrients (energy, protein, fat, calcium, phosphorus, NDF, NFC) were considered. Table 1

Table 1: Chemical compositions of feed

Feeds	Crude protein (g/kg)	NDF[1] (g/kg)	NFC[2] (g/kg)	Fat (g/kg)	Ca (g/kg)	P (g/kg)	NEl (Kcal/kg)	Price (Rial/kg)3	
Alfalfa	192	416	257	25	14.7	2.8	1190	5800	9000
Barley grain	124	208	617	22	0.6	3.9	1860	8000	9700
Sugar beet pulp	10	458	358	11	9.1	0.9	1470	8500	10500
Corn silage	88	450	387	32	2.8	2.6	1450	8250	10400
Cottonseed meal	449	308	157	19	2.0	11.5	1710	14500	17000
Fat supplement	0.0	0.0	0.0	845	120	0.0	5020	5000	7000
Sugar beet molasses	85	1.0	798	2.0	1.5	0.3	1840	2000	3500
Soybean meal	499	149	270	16	4.0	7.1	2130	17500	19500
Sunflower meal	284	403	222	14	4.8	10	1380	10500	12000
Wheat bran	173	425	296	43	1.3	11.8	1610	7100	8500
Oyster meal	0.0	0.0	0.0	0.0	380	0.0	0.0	20000	30000

[1] Neutral Detergent Fiber (NDF)
[2] Non Fibrous Carbohydrates (NFC)
[3] The feed price obtained from Mazandran Farming and Animal Husbandry Cooperative Union in June 2016.
Using T

shows the chemical composition of these materials:

Tables of nutritional requirements of dairy cattle (NRC) (2001), minimum and maximum nutritional requirements are obtained and used in fuzzy optimization problems solving. Table 2 shows the requirements and needs of lactating dairy cows in early lactation based on the minimum and maximum amount.

Interval and floating prices of nutrients are gathered from the market, converted to trapezoidal fuzzy numbers.

$$\tilde{c}_1 = (7000,7500,1200,1500)\,, \tilde{c}_2 = (8500,9100,500,600)\,,$$
$$\tilde{c}_3 = (9500,10000,1000,500)\,, c_4 = (9900,10000,750,400)\,,$$
$$\tilde{c}_5 = (16000,16500,1500,500)\,, \tilde{c}_6 = (5500,6500,500,500)\,,$$
$$\tilde{c}_7 = (2500,3000,500,500)\,, \tilde{c}_8 = (18000,18500,500,1000)\,,$$
$$\tilde{c}_9 = (11000,11500,500,500)\,, \tilde{c}_{10} = (7500,8000,400,500)\,,$$
$$\tilde{c}_{11} = (23000,25000,3000,5000).$$

Table 2: Nutritional requirements (kg/ dry matter) of lactating cows in early lactation, weighing between 600-700 kg

Feeds	Unit	Consistent model		
		Minimum	Maximum	Equivalent
Energy	Kcal/kg	1500	1650	
Protein	gr/kg	155	180	
Ether extract	gr/kg	30	80	
NDF	gr/kg	300	400	
NFC	gr/kg	350	420	
Calcium	gr/kg	10		
Phosphor	gr/kg	5		
Total carbohydrate	gr/kg		730	
Ratio of Ca: P	gr/kg			2
Total ration	gr/kg			1
Alfalfa hay	gr/kg		250	
Barley grain	gr/kg		300	
Sugar beet pulp	gr/kg		150	
Corn silage	gr/kg		150	
Cottonseed meal	gr/kg		120	
Fat supplement	gr/kg		40	
Sugar beet molasses	gr/kg		30	
Soybean meal	gr/kg		120	
Sunflower meal	gr/kg		100	
Wheat bran	gr/kg		150	
Oyster meal	gr/kg		25	

The fuzzy linear programming approach is a method that will be used to solve our research problem. So, we establish the Minimize Cost Diet problem (MCD) model as follows below:

$$MinZ = \tilde{c}_1 x_1 + \tilde{c}_2 x_2 + \tilde{c}_3 x_3 + \tilde{c}_4 x_4 + \tilde{c}_5 x_5 + \tilde{c}_6 x_6 + \tilde{c}_7 x_7 + \tilde{c}_8 x_8 + \tilde{c}_9 x_9 + \tilde{c}_{10} x_{10} + \tilde{c}_{11} x_{11}$$

subjec to

$$1190\,x_1 + 1860\,x_2 + 1470\,x_3 + 1450\,x_4 + 1710\,x_5 + 5020\,x_6 + 1840\,x_7 + 2130\,x_8 + 1380\,x_9 + 1610\,x_{10} \geq 1500$$

$$1190x_1 + 1860x_2 + 1470x_3 + 1450x_4 + 1710x_5 + 5020x_6 + 1840x_7 + 2130x_8 + 1380x_9 + 1610x_{10} \leq 1650$$

$$192x_1 + 124x_2 + 10x_3 + 88x_4 + 499x_5 + 85x_7 + 499x_8 + 284x_9 + 173x_{10} \geq 155$$

$$192x_1 + 124x_2 + 10x_3 + 88x_4 + 499x_5 + 85x_7 + 499x_8 + 284x_9 + 173x_{10} \leq 180$$

$$25x_1 + 22x_2 + 11x_3 + 32x_4 + 19x_5 + 845x_6 + 2x_7 + 16x_8 + 14x_9 + 43x_{10} \geq 30$$

$$25x_1 + 22x_2 + 11x_3 + 32x_4 + 19x_5 + 845x_6 + 2x_7 + 16x_8 + 14x_9 + 43x_{10} \leq 80$$

$$416x_1 + 208x_2 + 458x_3 + 450x_4 + 308x_5 + x_7 + 149x_8 + 403x_9 + 425x_{10} \geq 300$$

$$416x_1 + 208x_2 + 458x_3 + 450x_4 + 308x_5 + x_7 + 149x_8 + 403x_9 + 425x_{10} \leq 400$$

$$257x_1 + 617x_2 + 358x_3 + 378x_4 + 157x_5 + 798x_7 + 270x_8 + 222x_9 + 296x_{10} \geq 350$$

$$257x_1 + 617x_2 + 358x_3 + 378x_4 + 157x_5 + 798x_7 + 270x_8 + 222x_9 + 296x_{10} \leq 430$$

$$14.7x_1 + 0.6x_2 + 9.1x_3 + 2.8x_4 + 2x_5 + 120x_6 + 1.5x_7 + 4x_8 + 4.8x_9 + 1.3x_{10} + 380x_{11} \geq 10$$

$$2.8x_1 + 3.9x_2 + 0.9x_3 + 2.6x_4 + 11.5x_5 + 0.3x_7 + 7.1x_8 + 10x_9 + 11.8x_{10} \geq 5$$

$$673x_1 + 825x_2 + 816x_3 + 837x_4 + 465x_5 + 799x_7 + 419x_8 + 625x_9 + 721x_{10} \leq 730$$

$$9.1x_1 - 7.2x_2 + 7.3x_3 - 2.4x_4 - 21x_5 + 120x_6 + 0.9x_7 - 10.2x_8 - 15.2x_9 - 22.3x_{10} + 380x_{11} = 0$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} = 1$$

$x_1 \leq 0.25$, $x_2 \leq 0.30$, $x_3 \leq 0.15$, $x_4 \leq 0.15$, $x_5 \leq 0.12$, $x_6 \leq 0.04$, $x_7 \leq 0.03$, $x_8 \leq 0.12$, $x_9 \leq 0.10$, $x_{10} \leq 0.15$, $x_{11} \leq 0.025$

$x_j \leq 0$, $j = 1, 2, \ldots, 11$

By the use of these values in model and using the software GAMS and using the fuzzy primal simplex algorithm which is proposed in (Nasseri et al., 2009), the optimal solution is given in the below.

Table 3 : The quantities of food in one kg of diet rations

Feed ingredients	The quantity of food in diet
Alfalfa hay (gr)	250
Barley grain (gr)	166
Sugar beet pulp (gr)	117
Corn silage (gr)	150
Cottonseed meal (gr)	120
Fat supplement (gr)	13
Sugar beet molasses (gr)	27
Soybean meal (gr)	0
Sunflower meal (gr)	0
Wheat bran (gr)	150
Oyster meal (gr)	7

Table 4: Supplied requirement quantities of feed rations

Nutrient requirements	The amount of requirement that met
NEl (Kcal/kg)	1557.3
Protein (gram)	171.1
Ether extract (gram)	35.78
Neutral detergent fiber (gram)	300
Non Fibrous carbohydrate (gram)	350
Calcium (Ca) (gram)	10
Phosphor (P) (gram)	5
Price (Rial/kg)	9312.5

CONCLUSION

One of the pillars in aquaculture farming industries is formulation of food for the animals, which is also known as feed mix or diet formulation. However, the feed component in the aquaculture industry incurs the most expensive operational cost and has drawn many studies regarding diet formulation. The lack of studies involving modelling approaches had motivated to embark on diet formulation, which searches for the best combination of feed ingredients while satisfying nutritional requirements at a minimum cost. The result of model solving is given in Table 3, in which the cost of one kg diet is achieved as 9312.5. However, in linear programming models, it is necessary to consider certain. However, in sale market, prices are constantly faced changing and fluctuating, and therefore diet planners forcibly change diets according to price volatility. However, by considering safety margin for food price fluctuations, it can be possible to plan with more flexibility and confidence. Adjust the feed ration by the linear programming model is done with actual data. Although it is irrational to assume that there is often complete and accurate information about the data and needs and food prices used in the problem. Therefore fuzzy optimization method with floating price has been recommended to more accurate formulation of nutrient needs and feed amounts. According to the concepts of fuzzy sets and numbers used in this method, diet formulation will be more profitable and realistic. By this method, it can be possible to formulate cheaper and more suitable rations. Although this method is used in the dairy food ration formulation, it can also be applied to other kinds of diet formulation. Also providing software where fuzzy optimization method is used in animal feed rations, formulation will be helpful for the work of writers and researchers.

REFERENCE

Bas, E. (2014). A robust optimization approach to diet problem with overall glycemic load as objective function. *Applied Mathematical Modelling, 38*(19), 4926-4940.

Bazaraa, M.S., Jarvis, J.J., & Sherali, H.D. (2011). *Linear programming and network flows*. John Wiley & Sons.

Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management science, 17*(4), B-141.

Cadenas, J.M., Pelta, D.A., Pelta, H. R., & Verdegay, J. L. (2004). Application of fuzzy optimization to diet problems in Argentinean farms. European *Journal of Operational Research, 158*(1), 218-228.

Castrodeza, C., Lara, P., & Peña, T. (2005). Multicriteria fractional model for feed formulation: economic, nutritional and environmental criteria. *Agricultural systems, 86*(1), 76-96.

Ebrahimnejad, A., & Nasseri, S. H. (2009). Using complementary slackness property to solve linear programming with fuzzy parameters. Fuzzy *Information and Engineering, 1*(3), 233-245.

Ebrahimnejad, A., & Nasseri, S.H. (2010). A dual simplex method for bounded linear programmes with fuzzy numbers. *International Journal of Mathematics in Operational Research, 2*(6), 762-779.

Ebrahimnejad, A., Nasseri, S.H., Hosseinzadeh Lotfi, F., & Soltanifar, M. (2010a). A primal dual method for linear programming problems with fuzzy variables. *European Journal of Industrial Engineering, 4*(2), 189-209.

Ebrahimnejad, A., Nasseri, S.H., & Hosseinzadeh Lotfi, F., (2010b). Bounded linear programs with trapezoidal fuzzy numbers. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 18*(3), 269-286.

Ebrahimnejad, A., Nasseri, S.H., & Mansourzadeh, S.M., (2011). Bounded primal simplex algorithm for bounded linear programming with fuzzy cost coefficients. *International Journal of Operations Research and Information Systems, 2*(1), 96-120.

Ganesan, K., & Veeramani, P. (2006). Fuzzy linear programs with trapezoidal fuzzy numbers. *Annals of Operations Research, 143*(1), 305-315.

Gupta, R., & Chandan, M. (2013). Use of controlled random search technique for global optimization in animal diet problem. *International Journal of Emerging Technology and Advanced Engineering, 3*(2), 284-284.

Hsu, K.C. & Wang, F.S. (2013). Fuzzy optimization for detecting enzyme targets of human uric acid metabolism. *Bioinformatics, 29*(24), 3191–3198.

Iskander, M. G. (2006). A fuzzy weighted additive approach for stochastic fuzzy goal programming. *Applied Mathematics and Computation, 154*(2), 543-553.

Mahdavi-Amiri, N., & Nasseri, S. H. (2006). Duality in fuzzy number linear programming by use of a certain linear ranking function. *Applied Mathematics and Computation, 180*(1), 206-216.

Mahdavi-Amiri, N., & Nasseri, S. H. (2007). Duality results and a dual simplex method for linear programming problems with trapezoidal fuzzy variables. *Fuzzy sets and systems, 158*(17), 1961-1978.

Mahdavi-Amiri, N., Nasseri, S. H., & Yazdani, A. (2009). Fuzzy primal simplex algorithms for solving fuzzy linear programming problems. *Iranian Journal of Operations Research, 1*(2), 68-84.

Mamat, M., Rokhayati, Y., Mohamad, N.N., & Mohd, I. (2011). Optimizing human diet problem with fuzzy price using fuzzy linear approach.

Pakistan Journal of Nutrition, 10(6), 594-598.

Mamat, M., Deraman, S.K., Noor, N.M.M., & Mohd, I. (2012). Diet problem and nutrient requirement using fuzzy linear programming approaches. *Asian Journal of Applied Sciences, 5*(1), 52-59.

Maleki, H.R. (2002). Ranking functions and their applications to fuzzy linear programming, 283–301.

Maleki, H. R., Tata, M., & Mashinchi, M. (2000). Linear programming with fuzzy variables. *Fuzzy sets and systems, 109*(1), 21-33.

Marinov, E. & Daumalle, E. (2014). Index matrix interpretation and intuitionistic fuzzy estimation of the diet problem. *18th International Conference on Intuitionistic fuzzy sets system, Notes on Intuitionistic Fuzzy Sets, 20*(2), 75–84.

Moraes, L.E., Wilen, J.E., Robinson, P. H., & Fadel, J.G. (2012). A linear programming model to optimize diets in environmental policy scenario. *Journal of Dairy Science, 95*(3), 1267-1282.

Nasseri, S. H. (2008). A new method for solving fuzzy linear programming by solving linear programming. *Applied mathematical sciences, 2*(50), 2473-2480.

Nasseri, S.H., & Ebrahimnejad, A. (2010a). A fuzzy dual simplex method for fuzzy number linear programming problem. Advances in *Fuzzy Sets and Systems, 5*(2), 81-95.

Nasseri, S.H., & Ebrahimnejad, A. (2010b). A fuzzy primal simplex algorithm and its application for solving flexible linear programming problems. *European Journal of Industrial Engineering, 4*(3), 372-389.

Nasseri, S. H., & Mahdavi-Amiri, N. (2009). Some duality results on linear programming problems with symmetric fuzzy numbers. *Fuzzy Information and Engineering, 1*(1), 59-66.

NATIONAL RESEARCH COUNCIL. 2001: Nutrient Requirements of dairy cattle.

Niemi, J. K., Sevón-Aimonen, M. L., Pietola, K., & Stalder, K. J. (2010). The value of precision feeding technologies for grow–finish swine. *Livestock science, 129*(1), 13-23.

Piyaratne, M.K.D.K., Dias, N.G.J., & Attapattu, N.S.B.M. (2012). Linear model based software approach with ideal amino acid profiles for least-cost poultry ration formulation. *Information Technology Journal, 11*(7), 788-793.

Pomar, C., Dubeau, F., Létourneau-Montminy, M. P., Boucher, C., & Julien, P. O. (2007). Reducing phosphorus concentration in pig diets by adding an environmental objective to the traditional feed formulation algorithm. *Livestock Science, 111*(1), 16-27.

Saxena, P. (2011). Comparison of linear and nonlinear programming techniques for animal diet. Applied *Mathematics, 1*(2), 106-108.

SalooKolayi, D.D., Yansari, A.T., & Nasseri, S. H. (2011). Application of fuzzy optimization in diet formulation. *The Journal of Mathematics and Computer Science, 2*(3), 459-468.

Tanaka, H., Okuda, T., & Asai, K. (1974). On fuzzy mathematical programming. *The Journal of Cybernetics, 3*, 37-46.

Wang, F.S., Wu, W.H., & Hsu, K.C. (2014). Fuzzy optimization in metabolic systems. *International Journal of Biological, Food, Veterinary and Agricultural Engineering, 8*(7), 661-665.

Yager, R. R. (1981). A procedure for ordering fuzzy subsets of the unit interval. *Information sciences, 24*(2), 143-161.

Zhang, G., Wu, Y.H., Remias, M., & Lu, J. (2003). Formulation of fuzzy linear programming problems as four-objective constrained optimization problems. Applied Mathematics and Computation, 139(2), 383-399.

Zimmermann, H. J. (1987). Fuzzy set, decision making, and expert systems, International Series in Management Science. *Operations Research, Kluwer Academic Publishing, Boston*.

Zimmermann, H.J. (2001). Fuzzy set theory and applications. 4rd ed. Kluwer Academic Publisher.

Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy sets and systems, 1*(1), 45-55.