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A Combined Evaluation Method to Rank Alternatives Based on VIKOR and DEA with BELIEF Structure under Uncertainty

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Abstract

This paper processes a combined method, based on VIKOR and Data Envelopment Analysis (DEA) to select the units with most efficiency. We utilize the VIKOR as compromise solution method. This research is a two-stage model designed to fully rank the alternatives, where each alternative has multiple inputs and outputs. The problem involves BELIEF parameters in the solution procedure. First, the alternative evaluation problem is formulated by Data Envelopment Analysis (DEA) and in the second stage; we use the opinion of experts with belief parameter to be applied into a model of group Decision-Making (DM) called the aggregated fuzzy belief decision matrix. . In this method weight of efficiency witch obtained from VIKOR is multiply with weight of CCR model of data envelopment analysis. Finally, to illustrate the proposed method, an illustrative example is provided.

Keywords:
MCDM
VIKOR
BELIEF STRUCTURE
DEA

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INTRODUCTION

In real world the decision maker encounters circumstances in which variables and parameters are in uncertainty environment, and can`t determine the exact quantities for each variable, and this criticizes the accuracy and correctness of model. In this circumstance we need the model that could assess the efficiency of decision- making units noticing the data being uncertain. For example, the expert opinion about an alternative in relation to a given index is an average value. In the crisp state, a crisp number and in the fuzzy state, a fuzzy number can be attributed to this average value. Now the question is that whether there is any method through which an expert can incorporate all states (good, average, poor,…); let us say, in his view, the value of the given alternative relative to a specific index of 20% good, 70% average, and 30% poor. This is called the belief structure. Based on this evaluation view, multi-evaluation grades can be defined in decision making problems (Yang et al., 1994؛ Wang et al., 2006; Wei et al., 2009; Fu et al., 2010؛ Jiang et al., 2011)

In recent years, Jiang used this structure in decision-making problems and extended TOPSIS method in belief environment (Jiang et al., 2011). Furthermore, each of the states mentioned by the expert in this structure can be fuzzy; for example, in the above case where all states are expressed together with their percentages, a fuzzy number can be attributed to each of the good, average, and poor states. The number of levels used for expressing an expert's opinion about alternatives is optional; hence, the belief structure can be considered as the generalized form of fuzzy problem. Such structure is very practical in many real world problems and is closer to an expert's real opinion (Yang, 2001; Yang et al., 2002). Also, several experts can express their opinions about an issue; decision-making can be done in a group. In this regard, it has been tried to make the expert opinion as close to the quantified numerical value as possible. The belief method was used by Jiang in 2011 for ranking alternatives through TOPSIS method (Jiang et al., 2011).

VIKOR is one of the classical MADM methods of decision techniques and it was proposed by Opricovic (1998), and is regarded as an efficient tool to find a compromise solution emerging out of a set of conflicting criteria (Qin et al., 2015).It is an agreement multi attribute decision method and part of multi criteria decision-making, that was developed based on L.P metric method by Aprkovich and Zang. In this method, decision maker takes a VIKOR coefficient to create balance between L.P metric method when $p=\infty$ and when $p=1$.

Compromise methods are considered as raking alternatives in MADM problems, but in the present study, the aim is to utilize compromise decision-making methods in approximating the optimal answer in MADM programming problems.

LITERATURE REVIEW

Data Envelopment Analysis (DEA) measuring the relative efficiency of peer decision-making units (DMUs) with multiple inputs and multiple outputs was introduced by (Chrnse et al., 1978).

Data Envelopment Analysis (DEA) as a popular method has been extensively used for ranking and classifying the decision-making units. DEA deals with classifying the units into two categories, efficient and inefficient, based on two sets of multiple outputs contributing positively to the overall (Ganley et al.,1992; De et al., 2001). The ratio of weighted inputs and outputs produces a single measure of productivity called relative efficiency. The DMUs that have a ratio of 1 are referred to as "efficient", given the required inputs and produced outputs. The units that have a ratio less than 1 are "less efficient" relative to the most efficient units. Because the weights for the input and the output variables of DMU's are computed to maximize the ratio and then compared to a similar ratio of the best-performing DMU's, the measured productivity is also referred to as "relative efficiency"(Rouyendegh et al., 2010). The original DEA does not perform full ranking; it merely provides classification into two dichotomy groups: efficient and inefficient. It does not rank them; all efficient units are equally good in the Paratoo sense. For better ranking, researchers started to use of Multi-Criteria Decision-Making (MCDM) method. MCDM is a useful technique for determining the best solution among potential alternatives versus multiple criteria with different effects. However, the MCDM literature was entirely separate from DEA research until 1988, when Golany combined interactive, multiple-objective linear programming and DEA. Daneshvar proposed a hybrid model based on TOPSIS and Data Envelopment analysis (DEA) to evaluate the DMUs (Daneshvar rouyendegh, 2011).

Over the recent decades, other MCDM methods have been presented such as PROMETHEE and ELECTRE. These MCDM methods differ in many theoretical background and type of results given (Erahimnejad et al., 2012). Some MCDM methods have been constructed especially for one specific problem, and they are not suitable for other decision problems. For detail discussions, readers are referred to Ebrahimnejad et al. (2010, 2012) but the compromise solution is a feasible solution which is the closest to the ideal solution representing an agreement reached by mutual concessions. Two analytical multi-criteria techniques, namely TOPSIS and VIKOR, are often regarded as the well-known compromise solution methods for the MCDM problems. These methods are widely applied to numerous management and engineering fields (Mousavi et al., 2011; Vahdani et al., 2010).

Using compromise methods of decision-making is one of the most efficient methods for solving multi-objective programming problems on the basis of a compromise answer (Abo sinna et al., 2005). In recent years, some compromise decision-making methods with multiple indices have been utilized for solving large-scale problems with block angular structure (Abo sinna, 2000). In compromise methods which are mostly attributed to VIKOR and TOSIS methods, the value of answers depends upon their closeness to the positive ideal answer and their distance from the negative ideal answer, or from the regret (Abo sinna et al., 2005).

The VIKOR method was proposed to solve MCDM problems with conflicting and noncommensurable (different units) criteria, assuming that compromising is acceptable for conflict resolution, the decision maker wants a solution that is the closest to the ideal, and the alternatives are evaluated according to all established criteria" (Opricovice et al., 2007). Opricovic (1998) developed the initial VIKOR method. The VIKOR method is the optimization and compromise solution in MCDM, which is appropriate for estimating each alternative for each criterion (Opricovice et al., 2002; 2004; 2007; Huang et al., 2009). This method can be applied in the complex multicriteria system. The extended VIKOR method was developed and compared with TOPSIS, PROMETHEE, and ELECTRE .(Opricovice et al., 2007)

Mohaghar et al. used FAHP and the VIKOR method in selecting marketing strategy (Mohaghar et al., 2012).

Mandal et al believe the incorporation of fuzzy VIKOR technique enables us develop a ranking mechanism for the failure modes where the individual constituent components are non-commensurable in nature. The developed ranking mechanism helps the decision makers in optimal allocation of safety critical resources, used for risk mitigation purposes (Mandal et al., 2015).

Liau et al. proposed a new risk evaluation methodology for FMEA based on combination weighting and fuzzy VIKOR method to deal with the risk factors and identify the most serious failure modes for corrective actions (Liua et al., 2014).

Zhu ET AL. developed a systematic approach to manipulate the vagueness and subjectivity to enhance the objectivity in design concept evaluation by combining with rough number, analytic hierarchy process (AHP) and compromise ranking method (VIKOR) (Zhu et al., 2015).

In this research, the performance rating values of each alternative under the conflicting criteria as well as the weights of criteria are linguistic variables represented by Belief structure. Then, a new collective index is introduced to rank alternatives under a Belief structure. There are some advantages for the new our proposed method. Fuzzy evaluation grades and belief degrees are applied in the proposed multi-objective fuzzy belief structure model. Therefore, the experts can assign a more meaningful value for the coefficients considered based on the application of linguistic terms. Therefore, the expert knowledge and experience are combined since the proposed model is a group decision making model, we may use several experts' experiences. Moreover, we utilize an applicable relation to aggregate several experts' judgments.

MATERIALS AND METHODS

Belief structure

Belief structure can be proposed in n levels as:

$$S(C)=\{(H_n, \beta n) | \ n=1, 2, \ldots, N\} \quad (1)$$

Where H_n is the nth evaluation grade and β_n is a belief degree for its related grade. The kth expert proposes a belief structure number for each alternative versus criteria as:

$$S^k_{ij}=\{(H_n, b_{n,k}), \ n=1,\ldots,N\}_{ij} \quad (2)$$

$\beta_{n,k}$ is the feasibility percentage of n-th Level that is stated by k-th expert. Note that each evaluation grade can be proposed as a triangular or trapezoidal fuzzy number as shown in Fig. 1.

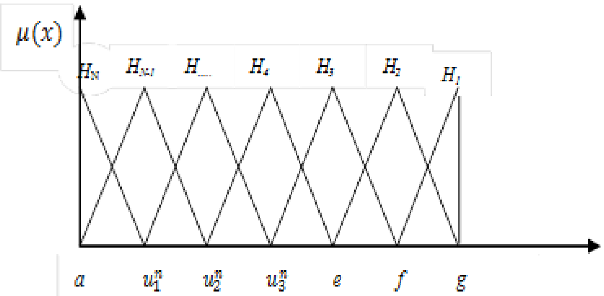

Fig.1. The mutual relationship between fuzzy evaluation grades

The vertical axe is mutual relationship and the horizontal axe is fuzzy evaluation grade.

Belief distance measure

According to definition in Eq. 1, a Fuzzy BS models consists of N fuzzy evaluation grades. To simplify the calculation, we firstly formulate a Fuzzy BS model as a corresponding vector

$B = v(S) = (b_1, b_2, \ldots, b_N)$. In this way, the comparison between two Fuzzy BS models is transformed into the distance measure between two vectors. Suppose there are two Fuzzy BS models S_1 and S_2, the corresponding vectors are B_1 and B_2. The distance between S_1 and S_2 is defined as:

$$d_{BS}(B_1,B_2)=\left(\frac{1}{2}(B_1-B_2)\hat{S}(B_1-B_2)^T\right)^{\frac{1}{2}} \quad (3)$$

Where $\hat{S}=\left[\hat{S}_{ij}\right]_{n\times n}$ is a similarity matrix, in which the element $\hat{S}_{ij}$ is defined in Eq. 4

$$\hat{S}_{ij}\left(H_i,H_j\right)=1-\frac{\sum_{k=1}^{4}\left|u_k^i-u_k^j\right|}{4} \quad (4)$$

In Eq. 4, $U(H_j)=(U_1^j,U_2^j,U_3^j,U_4^j)$ and $U(H_i)=(U_1^i,U_2^i,U_3^i,U_4^i)$ are two trapezoid fuzzy numbers. If U^i_2 and U^i_3 are equal then the number is converted into a fuzzy triangular number.

DEA method

Base model of data envelopment is CCR that is proposed as the basic CCR input -based (multiple) model:

$$\begin{aligned} &\max \sum_{r=1}^{s} u_r y_{rp} \\ &s.t. \ \sum_{i=1}^{m} v_i x_{ip} = 1 \\ &\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad ; j=1,2,\ldots,n \\ &u_r, v_i \ge 0 \quad ; r=1,2,\ldots,s \quad ,i=1,2,\ldots,m \end{aligned} \quad (5)$$

Where y_{ij} is the amount of r-th output for DMUj, x_{ij} is the amount of input i for DMUj, u_r is weight attached to output r and v_i is the weight attached to input i, n is the number of DMUs, s is the number of outputs, and m is number of inputs. Note that, the DMUs are used for the production of x_{ij} (in inputs) and y_{ij} (in outputs).

Vikor method

VIKOR method was introduced by Opricovic (1998). This method is an adaptive methods of compensatory models, So that the option would be preferred that is the nearest option to the ideal solution. In general, this method focuses on rankings and choosing options with a set of sometimes inconsistent indicators and finally provides adaptive solution that can help decision-makers reach the final solution. In this method indicators should be independent and qualitative indicators should be converted into quantitative indicators. For developed adaptive criteria (indicators) ranking in multi-criteria decision making, Lp-metric as an aggregation function is used in adaptive planning.

Suppose an MCDM problem that has m alternatives, $A_1 \ldots A_m$ and n criteria, $C_1 \ldots C_n$. Each alternative is assessed with respect to the n criteria. The rating of alternative *i* with respect to criterion *j* is denoted by f_{ij}, and the best and worst values are regarded as $f_j^- = \min_i f_{ij}$ and f_J^-, respectively. All the performance ratings assigned to the alternatives versus each criterion from a decision matrix are denoted by $X= (x_{ij})\ m \times n$. Let $W=(w_1,w_2,\ldots,w_n)$ be the relative weight vector about the criteria, satisfying $\sum_{j=1}^{n} w_j=1$ Then, the VIKOR method can be summarized as follows (Opricovice et al., 2002; 2004).

Step 1: Determine the best f_J^* and the worst f_J^- values of all criteria functions $j=1,2,\ldots,n$. If the *jth* Function represents a benefit then:

$$f_j^* = \max_i f_{ij} \tag{6}$$

$$f_j^- = \min_i f_{ij} \tag{7}$$

Step 2: compute the values S_i and R_i: $I=1, 2, \ldots, m$ by these relations:

$$S_i = \sum_{j=1}^{n} w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-) \tag{8}$$

$$R_i = \max_j w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-) \tag{9}$$

Where, W_{js} are the weights of criteria expressing their relative importance. Also, S_i expresses the individual regret/gaps, and R_i express the maximum individual gaps.

Step 3: compute the values Q_i: $i=1,2,\ldots,m$, by the following relation:

$$Q_i=v(S_i-S^*)/(S^- - S^*)+(1-v)(R_i-R^*)/(R^- - R^*) \tag{10}$$

Where

$$S^* = \min_i S_i, \quad S^- = \max_i S_i \tag{11}$$

$$R^* = \min_i R_i, \quad R^- = \max_i R_i \tag{12}$$

V is considered as the weight of the strategy of the majority of criteria or the maximum group utility the value of which is assumed to be v=0.5. Mini S_i emphasizes the minimization of the average sum of the individual regrets/gaps, and mini R_i represents the minimization of the maximum individual regret/gaps for prioritizing the improvement. In fact, *S* is the minimum value of Si, which is the maximum group utility, and *R* is the minimum value of R_i, which is the minimum individual regret of the opponent. Also, *Q* is the ranking index which is obtained based on the consideration of both group utility and individual regret of the opponent.

Step 4: Rank the alternatives. Sort them by values of *S, R* and *Q* in a decreasing order.

Proposed method

Step 1: in the first step, determine the evaluated result of e_k from DEA method based on CCR model.

Step 2: Aggregate the assessment of decision makers in fuzzy belief decision matrix with aggregated elements. All experts present all coefficients of objective functions and constraints with belief structure. $s_{ij}^k = \{(H_n, \beta_{nk}), n = 1, \ldots, N\}$ is a fuzzy BS model. It means that, the decision maker D_k believes that H_n is the evaluation indicator with belief degree β_{nk}. Therefore, the element S_{ij} is the aggregated judgment on coefficients of objective functions and constraints in the aggregated matrix. There is a relation between β_n and β_n kthat is expressed as:

$$\beta_n = \frac{\mu^* \left[\prod_{k=1}^{k} \left(w_K^D \beta_{n,k} + 1 - w_K^D \sum_{j=1}^{N} \beta_{j,k} \right) - \prod_{k=1}^{k} \left(1 - w_K^D \sum_{j=1}^{N} \beta_{j,k} \right) \right]}{1 - \mu^* \left[\prod_{K=1}^{K} \left(1 - W_K^D \right) \right]} \tag{13}$$

$$\mu = \left[\sum_{n=1}^{N} \prod_{k=1}^{k} \left(W_K^D \beta_{n,K} + 1 - w_K^D \sum_{j=1}^{N} \beta_{j,K} \right) - (N-1) \prod_{k=1}^{k} \left(1 - w_K^D \sum_{j=1}^{N} \beta_{j,K} \right) \right]^{-1} \tag{14}$$

With simultaneously considering the weight and position of experts we want to get to a view that represents all views. For this purpose the Eq. 13 and 14 are used. Where μ is mutual relationship and μ^* is the optimal mutual relationship. W^D_K is the weight of k-th indicator assigned by d-th decision maker.

Step 3: Determine the Positive Ideal Belief Solutions (PIBS) A^+ and Negative Ideal Belief Solutions (NIBS) A^-, respectively by:

$$f^+ = \{f_1^+, f_2^+, \ldots, f_j^+\} \quad (15)$$

$$f^- = \{f_1^-, f_2^-, \ldots, f_j^-\} \quad (16)$$

Step 4: Calculate the individual regret/gaps, and maximum individual gaps distance of positive ideal solution by Eq. 17-22.

$$S_i = \sum_{j=1}^{n} w_j \left(\frac{f_i^* - f_{ij}}{f_j^* - f_j^-} \right) = \sum_{j=1}^{n} w_j \left(\frac{d_{BS}(s_{ij}, s_j^+)}{d_{BS}(s_j^+, s_j^-)} \right) \quad i=1,2,\ldots,m \quad (17)$$

$$R_i = Max_j \left\{ w_j \left(\frac{f_i^* - f_{ij}}{f_j^* - f_j^-} \right) \right\} = Max_j \left\{ w_j \left(\frac{d_{BS}(S_{ij}, s_j^+)}{d_{BS}(s_j^+, s_j^-)} \right) \right\} \quad i=1,2,\ldots,m \quad (18)$$

Where d_{BS} in above formulas is calculated by Eq. 19

$$D_i^+ = \sqrt{\sum_{j=1}^{L} W_j^c d_{BS} \left(S_{ij}, S_j^+\right)^2}, \quad i=1,2,\ldots,m \quad (19)$$

Step 5: Calculate the relative closeness Q_i^{PIS} for each alternative A_i by Eq. 20

$$Q_i^{PIS} = v(S_i - S^*)/(S^- - S^*) + (1-v)(R_i - R^*)/(R^- - R^*) \quad (20)$$

Step 6: Calculate the regret and maximum distance of negative ideal solution by Eq. 21-22

$$S_i^{NIS} = \sum_{j=1}^{n} w_j \left(\frac{f_i^* - f_{ij}}{f_j^* - f_j^-} \right) = \sum_{j=1}^{n} w_j \left(\frac{d_{BS}(s_{ij}, s_j^+)}{d_{BS}(s_j^+, s_j^-)} \right) \quad i=1,2,\ldots,m \quad (21)$$

$$R_i^{NIS} = Max_j \left\{ w_j \left(\frac{f_i^* - f_{ij}}{f_j^* - f_j^-} \right) \right\} = Max_j \left\{ w_j \left(\frac{d_{BS}(S_{ij}, s_j^+)}{d_{BS}(s_j^+, s_j^-)} \right) \right\} \quad i=1,2,\ldots,m \quad (22)$$

Where d_{BS} in above formulas is calculated by Eq. 23

$$D_i^- = \sqrt{\sum_{j=1}^{L} W_j^c d_{BS} \left(S_{ij}, S_j^-\right)^2}, i = 1,2,\ldots,m \quad (23)$$

Step 7: calculate Q is the ranking index which is obtained based on the consideration of both group utility and individual regret of the opponent.

$$Q_i = v(S_i - S^*)/(S^- - S^*) + (1-v)(R_i - R^*)/(R^- - R^*) \quad (24)$$

Step 8: Calculate the final value of each alternative based on the result of e_k from DEA and of Q_i from new compromised method solution.

$$e'_k = e_k * R_K \quad (25)$$

RESULT AND DISCUSSION

In this section, we work out a numerical example to illustrate the new proposed method. The steps of the proposed decision making method are implemented in the application example from the literature. This application example has been evaluated by DEA methodology (Mehdiabadi et al., 2013). In this article, they first present details of their finding on the implementation of DEA technique on ranking 15 different industries. The inputs of their proposed DEA model include capital, employment and outputs are exports and added value that have shown in Table 1. After applying DEA method, they find efficient and inefficient units demonstrated in Table 2. In addition, there is more than one efficient unit, which creates motivation to use VIKOR compromise method in belief structure for ranking them.

Then the decision matrix is constructed in four levels. The levels are defined as:

(very high, high, average and low) Where (very high, high, average and low) are fuzzy numbers. Moreover, the amounts of decision matrix can result from several experts through the group decision making process. In other words, this problem is a group decision making problem.

This means that each coefficient is proposed by five decision makers: D_1, D_2, D_3, D_4 and D_5. Each judgment is expressed by FBS with four evaluation grades $\{H_1, H_2, H_3 \text{ and } H_4\}$ = {*'very high', 'high', 'average' and 'low'*}.

Where low=(0, 0, 0, 0.2), average = (0.1, 0.3, 0.5, 0.7), high = (0.5, 0.6, 0.7, 0.9), very high = (0.8, 1, 1, 1) and

$$\tilde{s} = \begin{bmatrix} 1.000 & 0.725 & 0.450 & 0.100 \\ 0.725 & 1.000 & 0.725 & 0.375 \\ 0.450 & 0.725 & 1.000 & 0.650 \\ 0.100 & 0.375 & 0.650 & 1.000 \end{bmatrix}$$

Table 1: The application example

	Capital	employment	export	Added value
Food industry	169322	770	3145111	126675
Garment	140336	454	47780020	24796
Wood and woody production	6612	109	0	5100
Paper production	45692	67	1175739	15869
Coke coal industries	76225	69	0	15118
Chemical Material Production	994032	448	86766279	562574
Rubber and Tier products	1883399	316	13199657	27940
Non-metallic mineral products	408908	1029	74458796	195017
Production of basic metals	119323	331	76318036	24961
Original metal products	43538	656	0	72948
Machinery and equipment products	41730	1038	144204	119848
Machines generating electricity transmission	99754	186	0	59427
Production of motor vehicles	194956	425	152163	20797
Production of other transport equipment	4845	120	0	14036
production of furniture and unclassified manufactures	12340	268	0	40054

Table 2: The results of the implementation of CCR method

DMUS	Efficiency	Efficient or non-efficient
DMU1	84.3%	
DMU2	85.6%	
DMU3	100%	✓
DMU4	99.7%	
DMU5	39.4%	
DMU6	100%	✓
DMU7	33.5%	
DMU8	95.2%	
DMU9	100%	✓
DMU10	82.3%	
DMU11	100%	✓
DMU12	100%	✓
DMU13	100%	✓
DMU14	100%	✓
DMU15	100%	✓

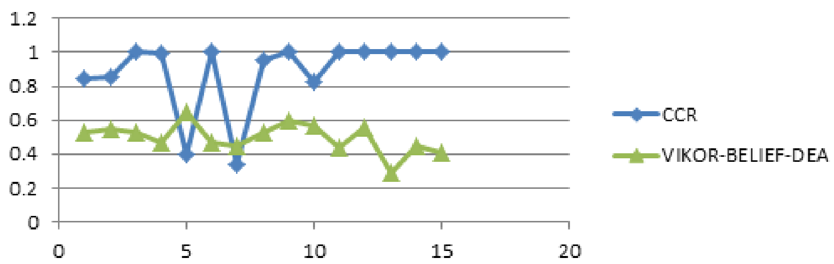

Fig. 2. The behavior of alternatives respect to CCR and VIKOR-DEA-Belief structure

Table 3: The decision matrix based on opinion of experts for respect to criterion Capital

Alternatives	Low	average	high	Very high
Food industry	0.1091	0.2666	0.4440	0.1803
Garment	0.1094	0.2875	0.4430	0.1601
Wood and woody production	0.0793	0.1764	0.5379	0.2064
Paper production	0.4218	0.1776	0.2283	0.2574
Coke coal industries	0.1390	0.1294	0.4311	0.3181
Chemical Material Production	0.6499	0.1471	0.1106	0.1720
Rubber and Tier products	0.6850	0.1197	0.1111	0.0841
Non-metallic mineral products	0.4644	0.3290	0.1084	0.0981
Production of basic metals	0.2435	0.1534	0.3875	0.2156
Original metal products	0.1688	0.3122	0.4080	0.1111
Machinery and equipment products	0.5364	0.2089	0.1376	0.1086
Machines generating electricity transmission	0.2840	0.4440	0.1711	0.1009
Production of motor vehicles	0.6521	0.1502	0.1126	0.0852
Production of other transport equipment	0.4783	0.2748	0.1384	0.1085
production of furniture and unclassified manufactures	0.4624	0.3272	0.1198	0.0907

Table 4: The opinion of experts for respect to criterion employment

	Low	average	high	Very high
Food industry	0.0886	0.1977	0.5267	0.1869
Garment	0.0708	0.1436	0.1938	0.5846
Wood and woody production	0.0956	0.1246	0.1834	0.5963
Paper production	0.2994	0.3877	0.1825	0.1304
Coke coal industries	0.0958	0.1249	0.1940	0.5852
Chemical Material Production	0.3996	0.2165	0.1633	0.2206
Rubber and Tier products	0.6740	0.1392	0.1023	0.0845
Non-metallic mineral products	0.4766	0.2639	0.1298	0.1298
Production of basic metals	0.0804	0.1385	0.4641	0.3170
Original metal products	0.0956	0.1243	0.1830	0.5971
Machinery and equipment products	0.2841	0.4785	0.1388	0.0988
Machines generating electricity transmission	0.2264	0.1958	0.1857	0.3746
Production of motor vehicles	0.6187	0.1719	0.1140	0.0955
Production of other transport equipment	0.1005	0.4562	0.1806	0.2628
production of furniture and unclassified manufactures	0.2628	0.5288	0.1173	0.1272

Table 5: The opinion of experts for respect to criterion export

	Low	average	high	Very high
Food industry	0.0791	0.1556	0.2276	0.5376
Garment	0.0873	0.1348	0.2051	0.5728
Wood and woody production	0.0707	0.1372	0.3163	0.4758
Paper production	0.3350	0.2178	0.2832	0.1640
Coke coal industries	0.0878	0.1356	0.2273	0.5493
Chemical Material Production	0.2907	0.1853	0.1747	0.3492
Rubber and Tier products	0.4330	0.1938	0.1625	0.2107
Non-metallic mineral products	0.1104	0.4544	0.2758	0.1595
Production of basic metals	0.0983	0.1473	0.5143	0.2401
Original metal products	0.0979	0.1564	0.2073	0.5383
Machinery and equipment products	0.1593	0.4697	0.2117	0.1594
Machines generating electricity transmission	0.1292	0.2115	0.1915	0.4677
Production of motor vehicles	0.5514	0.2068	0.1362	0.0972
Production of other transport equipment	0.1381	0.4945	0.1912	0.1763
production of furniture and unclassified manufactures	0.1321	0.5862	0.1934	0.0967

Table 6: The opinion of experts for respect to criterion added value

	Low	average	high	Very high
Food industry	0.0938	0.4204	0.2034	0.2669
Garment	0.0879	0.1446	0.2058	0.2617
Wood and woody production	0.1002	0.1703	0.4297	0.2998
Paper production	0.3366	0.3180	0.1637	0.1729
Coke coal industries	0.1002	0.1700	0.4417	0.2882
Chemical Material Production	0.3468	0.1641	0.3264	0.1627
Rubber and Tier products	0.6407	0.1606	0.1042	0.0945
Non-metallic mineral products	0.2845	0.4438	0.1611	0.1106
Production of basic metals	0.0878	0.1353	0.2157	0.5611
Original metal products	0.0876	0.1644	0.2629	0.1851
Machinery and equipment products	0.3027	0.1848	0.2843	0.2282
Machines generating electricity transmission	0.0805	0.1580	0.3075	0.4540
Production of motor vehicles	0.5959	0.1928	0.1337	0.0776
Production of other transport equipment	0.3217	0.3961	0.1615	0.1207
production of furniture and unclassified manufactures	0.3902	0.3686	0.1406	0.1007

Table 7: The obtained weight by VIKOR method

	VIKOR method
Food industry	0.6210
Garment	0.6423
Wood and woody production	0.5241
Paper production	0.4702
Coke coal industries	0.6432
Chemical Material Production	0.4679
Rubber and Tier products	0.4452
Non-metallic mineral products	0.5321
Production of basic metals	0.5999
Original metal products	0.5668
Machinery and equipment products	0.4329
Machines generating electricity transmission	0.5566
Production of motor vehicles	0.2897
Production of other transport equipment	0.4438
production of furniture and unclassified manufactures	0.4037

Table 8: Result of evaluating with proposed method

Alternatives	PROPOSED METHOD	CCR
Food industry	0.5235	0.843
Garment	0.5498	0.856
Wood and woody production	0.5241	1
Paper production	0.470	0.997
Coke coal industries	0.6432	0.394
Chemical Material Production	0.4679	1
Rubber and Tier products	0.4452	0.335
Non-metallic mineral products	0.5321	0.952
Production of basic metals	0.5999	1
Original metal products	0.5668	0.823
Machinery and equipment products	0.4329	1
Machines generating electricity transmission	0.5566	1
Production of motor vehicles	0.2897	1
Production of other transport equipment	0.4438	1
production of furniture and unclassified manufactures	0.4037	1

The decision matrix based on opinion of experts for respect to criterion Capital is proposed in Table 3. The opinion of experts for respect to criterion employment is proposed in Table 4. The opinion of experts for respect to criterion export is proposed in Table 5. The opinion of experts for respect to criterion added value is proposed in Table 6. The obtained weight by VIKOR compromise method is proposed in Table 7. In Table 8, result of ranking with proposed Solution is proposed in Table 8. The behavior of alternatives respect to CCR and VIKOR method-DEA-Belief structure is shown in Fig. 1.

As presented in Table 8, the second column shows the scores of the fifteen industries. The result score is always the bigger the better. Although there is no perfect compatibility between DEA and our new method in the general case, empirically, we found some examples of match units. Some of units that were efficient by DEA (unit 3, 6, 9 and 10-15) and we could not rank them but by new method we rank them.

CONCLUSION

In this paper, we have demonstrated a simple and easy-to-use method for department comparison via DEA. Furthermore, we integrated VIKOR solution and DEA under belief structure to generate a more feasible DEA result. We have presented an effective model for rank scaling of the units with multiple inputs and multiple outputs using both DEA and new method. In this paper, we focused on applying the belief structure as Intuitive concept to evaluate DMUs. Because new compromise method is applied in discrete decision making problems and DEA is applied in Continuous problem, therefore, we utilized the advantages of VIKOR method and DEA simultaneously. The results of DEA provided more than one efficient DMU and the applied method implemented VIKOR method technique to rank the efficient DMUs. On the other hand, in real-world situation decision making problems, the evaluating of alternatives usually cannot be assigned by single linguistic variables. Therefore, it is more proper that decision makers' judgments are assigned through multiple levels of factors with belief degrees. Therefore, the opinion of experts was proposed with belief structure. Our new method combined the concepts of VIKOR, DEA and belief structure to evaluate the alternatives versus many criteria. In other words, the proposed method applied the advantages of VIKOR, DEA and belief structure simultaneously. Finally, to justify the proposed method, an illustrative example was provided. The proposed method was applied as a group decision making problem to exploit many experts' judgments. Moreover, we utilized a mathematical relation to aggregate the experts' judgments which consider weights of experts and objective functions. For future research, the proposed problem may be solved by the other MCDM methods. This study can be extended for more sophisticated MCDM applications. For instance, we may consider uncertainty on input/output parameters and we leave it for interested researchers as future study.

REFERENCE

Abo-Sinna, M.A. (2000). Extensions of the TOPSIS for multi-objective dynamic programming problems under fuzziness. *Journal of Advances in Modeling and Analysis, 43* (4), 1–24.

Abo-Sinna, M. A., & Amer, A. H. (2005). Extensions of TOPSIS for multi-objective large-scale nonlinear programming problems. *Applied Mathematics and Computation, 162*(1), 243–256.

Charnes, A., Cooper, W.W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European journal of operational research, 2*(6), 429-444.

Cheng, E.W., Chiang, Y.H., & Tang, B.S. (2007). Alternative approach to credit scoring by DEA: Evaluating borrowers with respect to PFI projects. *Building and Environment, 42*(4), 1752-1760.

Daneshvar Rouyendegh, B. (2011). The DEA and intuitionistic fuzzy TOPSIS approach to departments' performances: a pilot study. *Journal of Applied Mathematics*, 2011.

De Koster, M. B. M., Balk, B. M., & Van Nus, W. T. I. (2009). On using DEA for benchmarking container terminals. *International Journal of Operations & Production Management, 29*(11), 1140-1155.

De, S.K., Biswas, R., & Roy, A.R. (2001). An application of intuitionistic fuzzy sets in medical diagnosis. *Fuzzy sets and Systems, 117*(2), 209-213.

Ebrahimnejad, S., Mousavi, S. M., & Seyrafianpour,

H. (2010). Risk identification and assessment for build–operate–transfer projects: A fuzzy multi attribute decision making model. *Expert systems with Applications, 37*(1), 575-586.

Ebrahimnejad, S., Mousavi, S.M., Tavakkoli-Moghaddam, R., Hashemi, H., & Vahdani, B. (2012). A novel two-phase group decision making approach for construction project selection in a fuzzy environment. *Applied Mathematical Modelling, 36*(9), 4197-4217.

Fu, C., & Yang, S. L. (2010). The group consensus based evidential reasoning approach for multiple attributive group decision analysis. *European Journal of Operational Research, 206*(3), 601-608.

Ganley, J. A., & Cubbin, J. S. (1992). *Public sector efficiency measurement: Applications of data envelopment analysis.* Elsevier Science Inc.

Golany, B. (1988). An interactive MOLP procedure for the extension of DEA to effectiveness analysis. *Journal of the Operational Research Society, 39*(8), 725-734.

Huang, J. J., Tzeng, G. H., & Liu, H. H. (2009). A revised VIKOR model for multiple criteria decision making-The perspective of regret theory. In Cutting-Edge Research Topics on Multiple Criteria Decision Making (pp. 761-768). *Springer Berlin Heidelberg*.

Jafarian-Moghaddam, A.R., & Ghoseiri, K. (2011). Fuzzy dynamic multi-objective Data Envelopment Analysis model. *Expert Systems with Applications, 38*(1), 850-855.

Jiang, J., Chen, Y.W., Chen, Y.W., & Yang, K.W. (2011). TOPSIS with fuzzy belief structure for group belief multiple criteria decision making. *Expert Systems with Applications, 38*(8), 9400-9406.

Jolai, F., Yazdian, S. A., Shahanaghi, K., & Khojasteh, M. A. (2011). Integrating fuzzy TOPSIS and multi-period goal programming for purchasing multiple products from multiple suppliers. *Journal of Purchasing and Supply Management, 17*(1), 42-53

Liu, H. C., You, J. X., You, X. Y., & Shan, M. M. (2014). A novel approach for failure mode and effects analysis using combination weighting and fuzzy VIKOR method. *Applied Soft Computing, 28*, 579-588.

Lozano, S., & Villa, G. (2004). Centralized resource allocation using data envelopment analysis. *Journal of Productivity Analysis, 22*(1), 143-161.

Mandal, S., Singh, K., Behera, R.K., Sahu, S. K., Raj, N., & Maiti, J. (2015). Human error identification and risk prioritization in overhead crane operations using HTA, SHERPA and fuzzy VIKOR method. *Expert Systems with Applications, 42*(20), 7195-7206.

Mehdiabadi, A., Rohani, A., & Amirabdollahiyan, S. (2013). Ranking industries using a hybrid of DEA-TOPSIS. *Decision Science Letters, 2*(4), 251-256.

Mohaghar, A., Fathi, M.R., Zarchi, M.K., & Omidian, A. (2012). A combined VIKOR-fuzzy AHP approach to marketing strategy selection. *Business Management and Strategy, 3*(1), 13.

Mousavi, S.M., Jolai, F., & Tavakkoli-Moghaddam, R. (2011). A fuzzy stochastic multi- attribute group decision-making approach for selection problems. Group Decis. Negot. http://dx. doi. org /10.1007/s10726-011-9259-1.

Opricovic, S. (1998). Multicriteria optimization of civil engineering systems. *Faculty of Civil Engineering, Belgrade, 2*(1), 5-21.

Opricovic, S., & Tzeng, G. H. (2002). Multicriteria planning of post-earthquake sustainable reconstruction. *Computer-Aided Civil and Infrastructure Engineering, 17*(3), 211-220.

Opricovic, S., & Tzeng, G. H. (2004). Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *European journal of operational research, 156*(2), 445-455.

Opricovic, S., & Tzeng, G. H. (2007). Extended VIKOR method in comparison with outranking methods. *European journal of operational research, 178*(2), 514-529.

Qin, J., Liu, X., & Pedrycz, W. (2015). An extended VIKOR method based on prospect theory for multiple attribute decision making under interval type-2 fuzzy environment. *Knowledge-Based Systems, 86,* 116-130.

Ramanathan, R. (2007). Supplier selection problem: integrating DEA with the approaches of total cost of ownership and AHP. *Supply Chain Management: an international journal, 12*(4), 258-261.

Rouyendegh, B.D., & Erol, S. (2010). The DEA–FUZZY ANP department ranking model applied in Iran Amirkabir University. *Acta Polytechnica Hungarica, 7*(4), 103-114.

Vahdani, B., Hadipour, H., Sadaghiani, J.S., & Amiri, M. (2010). Extension of VIKOR method based on interval-valued fuzzy sets. The International Journal of Advanced Manufacturing Technology, 47(9-12), 1231-1239.

Wang, T. C., Liang, J. L., & Ho, C. Y. (2006, October). Multi-criteria decision analysis by using fuzzy VIKOR. In *Service Systems and Service Management, 2006 International Conference on*, 2, 901-906, IEEE.

Wei, S. H., & Chen, S. M. (2009). A new approach for fuzzy risk analysis based on similarity measures of generalized fuzzy numbers. *Expert Systems with Applications, 36*(1), 589-598.

Yang, J. B., & Xu, D. L. (2002). On the evidential reasoning algorithm for multiple attribute decision analysis under uncertainty. IEEE Transactions on Systems, Man, and Cybernetics-Part A: *Systems and Humans, 32*(3), 289-304.

Yang, J. B., & Sen, P. (1994). A general multi-level evaluation process for hybrid MADM with uncertainty. *IEEE Transactions on Systems, Man, and Cybernetics, 24*(10), 1458-1473.

Yang, J. B. (2001). Rule and utility based evidential reasoning approach for multiattribute decision analysis under uncertainties. *European journal of operational research, 131*(1), 31-61.

Zhu, G. N., Hu, J., Qi, J., Gu, C. C., & Peng, Y. H. (2015). An integrated AHP and VIKOR for design concept evaluation based on rough number. *Advanced Engineering Informatics, 29*(3), 408-418.