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Alternative Ranking Method in Dynamic Data Envelopment Analysis (DDEA)

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Abstract

The motivation of this paper is to propose such equitable method for ranking all decision making units (DMUs) in dynamic Data Envelopment Analysis (DDEA) framework. As far as we are aware there is not more studies in dynamic DEA literature. What's more, in such cases the best operating unit is important to be sampled for the others in under evaluated time periods. However, in this special concept of DEA, quasi-fixed inputs or intermediate products are the source of inter temporal dependence between consecutive periods. Hence, in order to have suitable ranking for units operating in dynamic environment the minimum and maximum efficiency values of each DMU in dynamic state are computed. Also, we assume that the sum of efficiency values of all DMUs in dynamic state is equal to unity. Thereafter, the rank of each DMU is determined through the combination of its maximum and minimum efficiency values. A real case of Iranian gas companies highlights the applicability of the proposed method in Dynamic framework.

Keywords:
efficiency
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INTRODUCTION

Data Envelopment Analysis (DEA) is a well-known non- parametric technique to assess the relative performance of a set of homogeneous decision making units (DMUs) with multiple incommensurate inputs and outputs. DEA was developed by Charnes, Cooper and Rhodes (1978) (CCR model) in constant returns to scale environment and then it has been extended by Banker, Charnes and Cooper (1984) (BCC model) to variable returns to scale environment. Efficiency measurement in DEA requires approaches consistent with the underlying technology. In the last four decades, this methodology has been widely applied in different areas, such as banking, healthcare, supply-chain management, and so on. An interesting subject study in DEA is the existence of inter-connecting activities. Considering the fact these activities can play important role in many real cases, so the development of technologies with these connections is an important subject of concern in the production process. Fare and GrossKopf (1996) pioneered the first innovative scheme for dealing with these activities in DEA framework. In fact, their dynamic DEA framework was originated to cope with long-time assessment incorporating the concepts of quasi-fixed inputs and investment activities. The proposed dynamic DEA (DDEA) format enables us measuring the efficiency based upon the long time optimization in which inter-connecting activities such as investment activities are considered. This feature of dynamic DEA discriminates from the separate time models such as Window analysis (KLOPP,1985) and Malmquist productivity index (Malmquist, 1953). Examples of other views in dynamic DEA (DDEA) literature can be found in Nemoto and Goto (1999), Sueyoshi and Sekitani (2005), Tone and Tsutsui (2009) and Amirteimoori (2006) As far as we are aware, an equitable method for ranking DMUs in dynamic DEA framework has not been developed. Toward this end, this paper applies the ranking method of Khodabakhshi and Aryavash (2012) in dynamic DEA literature. In order to design new ranking method in dynamic framework, it is assumed that the sum of efficiency values of all DMUs is equal to unity. Also, the minimum and maximum efficiency values of all DMUs are computed. Finally, the rank of each DMU is determined through the combination of its minimum and maximum efficiency values. The paper is organized as follows. In the following section a brief review of dynamic DEA (DDEA) method is presented. An alternative ranking method in dynamic DEA (DDEA) framework will be presented in Section 3. Section 4 applies the method to an example involving 11 Iranian gas companies in two periods. Conclusion will end the paper.

DYNAMIC DEA (DDEA)

As far as we are aware in all previous standard DEA literature all inputs and outputs are regarded as real-valued quantities. The introduction of different type of inputs or quasi-fixed inputs can be seen as a first step toward dynamic DEA. A unique feature of the quasi-fixed inputs is that those are considered as outputs at the current period, while being treated as inputs at the next period. For example, in power generators, workers and fuels as variable inputs are employed to generate electricity as outputs. Most of the generated power is sold to the purchasers, though a part of the generated power (as quasi-fixed input) is internally saved within the generator. This saved power is used to generate electricity in the next period. So, it applies as quasi-fixed input. The first literature effort in Dynamic DEA (DDEA) refers to Nemoto and Goto (1999). In their research two different types of inputs are considered: variable inputs and quasi-fixed inputs. Assume that n DMUs $(j=1,...,n)$ are examined in T periods. In period t, each unit DMUj uses two different groups of inputs: $k_j^{(t-1)} \in R^1_+$ as a vector of quasi-fixed inputs and $x^{(t)}_j \in R^m_+$ a vector of variable inputs to produce two types of outputs: $y^{(t)}_j \in R^s_+$ as a

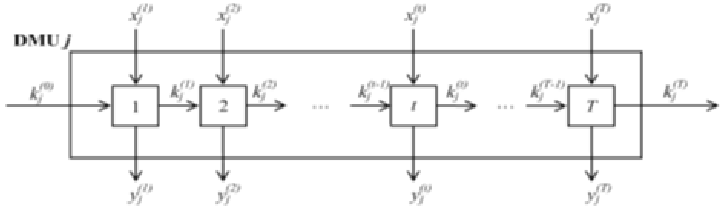

Fig. 1. Dynamic DEA Structure

vector of goods and $k^{(t)}_j$ as the vector of quasi-fixed inputs used in the next period. Dynamic DEA (DDEA) structure is illustrated in Fig. 1.

As the figure shows variable inputs $x^{(t)}_j$ and quasi-fixed inputs $k^{(t-1)}_j$ at the beginning of period t are transformed by the process P_t into regular outputs $y^{(t)}_j$ and quasi-fixed inputs $k^{(t)}_j$ at the end of period t. Based on defined postulate the production possibility set (PPS) in period t can be rewritten as follows:

$$\phi_t^{CRS} = \{(x^{(t)}, k^{(t-1)}, y^{(t)}, k^{(t)}) \in R_+^{l+m} \times R_+^{s+l}$$

$$\begin{vmatrix} X_t\lambda_t \le x^{(t)} & K_{t-1}\lambda_t \le k^{(t-1)} \\ Y_t\lambda_t \ge y^{(t)} & K_t\lambda_t \ge k^{(t)} \end{vmatrix} , \lambda_t \ge 0\}$$

Where $\lambda_t \in R^n_+$ is the intensity weight components which connect DMUs in period t. Also, $X_t=[x^{(t)}_1,...,x^{(t)}_n)]$, $K_{t-1}=[k^{(t-1)}_1,...,k^{(t-1)}_n]$ and $Y_t=[y^{(t)}_1,…,y^{(t)}_s]$ are the matrices of variable inputs, quasi-fixed inputs and outputs respectively. Let DMU_0 be under evaluation unit which uses ($x^{(t)}_0$, $k^{(t-1)}_0$) to produce ($y^{(t)}_0$, $k^{(t)}_0$) for $t=1,2,..,T$. now the question is how to rank the units in Dynamic DEA (DDEA) framework. The next section propose the ranking method which is our basement for ranking dynamic structures.

THE PROPOSED RANKING METHOD IN DYNAMIC DEA (DDEA)

In this section we extend the ranking method proposed by Khodabakhski and Aryavash (2012) in Dynamic DEA (DDEA) framework. As before assume that there are n DMUs $(j=1,...,n)$ over T periods $(t=1,...,T)$. Let DMU_0 be under evaluation unit that converts ($x^{(t)}_0$, $k_0^{(t-1)}$) into produce ($y^{(t)}_0$, $k_0^{(t)}$) in period t $(t=1,...,T)$. Without less of generality suppose that the number of variable equal to unity i.e., $m=l=s=1$. Let $x^{(t)}_j$ and $y^{(t)}_j$ denote the input and output of j-th DMU in period $(t=1,...,T)$. Also, $X_j=\sum_{t=1}^{T} x_j^{(t)}$ and $Y_j=\sum_{t=1}^{T} y_j^{(t)}$ denote the total quantities of the input and output respectively, over all T periods. As (Khodaakhshi et al., 2012) proposed, in order to estimate the efficiency value of $DMU_0(\theta_0)$, assume that the sum of efficiency of all DMUs equal to unity $(\sum_{j=1}^{n}\theta_j=1)$.

As usual the weighted sum of outputs divided by the weighted sum of inputs is defined as the efficiency of DMUs. So, the weight vectors $u=(u_1,...,u_s)$, $v=(v_1,...,v_m)$ and $w=(w_1,...,w_l)$ are assigned to inputs, outputs and quasi-fixed inputs respectively. According to Kao (2013) the overall efficiency of $DMU_j(\theta_j)$. And period efficiency of $DMU_j(\theta^{(t)}_j)$ can be determined using the following Equations:

$$\theta_j = \frac{uY_j + wk_j^{(T)}}{vX_j + wk_j^{(o)}} \qquad j=1,...,n \qquad (2)$$

$$\theta_j^{(t)} = \frac{uy_j^{(t)} + wk_j^{(t)}}{vx_j^{(t)} + wk_j^{(t-1)}} \qquad j=1,...,n \qquad (3)$$

Following Khodabakhshi and Arya vash (2012), Eq. 2 and 3 cannot be used to compute the unique values for (θ_j) but they can be used to determine their minimum and maximum values as follows:

$$\begin{aligned} &\min \text{ and } \max \quad \theta_o \\ &s.t. \\ &\theta_j \le 1 \qquad j=1,...,n \\ &\theta_j^{(t)} \le 1 \qquad j=1,...,n \quad ,t=1,...,T \qquad (4) \\ &\sum_{j=1}^{n}\theta_j = 1 \end{aligned}$$

Substituting Eq. 2 and 3 in above model, the model must be run twice. The minimum value of $\theta_0(\theta_0^{\min})$ is determined by minimizing the objective function of model (4). Also, the maximum value of is determined by maximizing the objective function of model (4).This model is a nonlinear program, so we transform it into a linear form. We now rewrite the fractional format of model (4) as follows:

$$\theta_j = \frac{uY_j + wk_j^{(T)}}{vX_j + wk_j^{(o)}} \Rightarrow uY_j + wk_j^{(T)} - [(v\theta_j)X_j + (w\theta_j)k_j^{(o)}] = 0 \quad j=1,...,n \qquad (5)$$

Using the transformation $v\theta_j = \overline{v}_j$ and $w\theta_j = \overline{w}_j$, model (4) can be replaced by the following linear programming problem:

$$\min \quad and \quad \max \quad \theta_o = uY_o + wk_o^{(T)}$$
$$s.t.$$
$$vX_o + wk_o^{(o)} = 1$$
$$uY_j + wk_j^{(T)} - (vX_j + wk_j^{(o)}) \le 0$$
$$j = 1,\ldots,n,$$
$$uy_j^{(t)} + wk_j^{(t)} - (vx_j^{(t)} + wk_j^{(t-1)}) \le 0$$
$$j = 1,\ldots,n, \quad t = 1,\ldots,T,$$
$$uY_j + wk_j^{(T)} - (\bar{v}_j X_j + \bar{w}_j k_j^{(o)}) = 0$$
$$j = 1,\ldots,n,$$
$$\sum_{j=1}^{n} \bar{v}_j = v,$$
$$\sum_{j=1}^{n} \bar{w}_j = w,$$
$$u, v, w \ge 0 \quad \bar{v}_j, \bar{w}_j \ge 0$$
$$j = 1,\ldots,n,$$

Note that the sum of the constraints in the third constraint set, i.e., the constraints associated with the periods, is equal to the constraint in the second constraint set, i.e., the constraints associated with the system, for each DMU. Therefore, the second constraint is redundant and can be deleted. Model (6) always has a feasible solution. Then the implementation of model (6) the minimum and maximum values of θ_j are obtained.

Theorem 1: if ($\theta_j^{\min}$) and ($\theta_j^{\max}$) are optimal solutions of model (6) for each DMU_j then $\theta_j^{\min} \le \theta_j^{\max}$.

Proof: the proof is straightforward with respect to model (6).

Hence, we have the following interval for each θ_j:

$$\theta_j^{\min} \le \theta_j \le \theta_j^{\max} \qquad j=1,\ldots,n \tag{7}$$

Following the strategy proposed in (Khodaakhshi et al., 2012) θ_j can be rewritten as the convex combination of minimum and maximum amount of efficiency:

$$\theta_j = \lambda_j \theta_j^{min} + (1-\lambda_j)\, \theta_j^{max}, \quad 0 \le \lambda_j \le 1, \quad j=1,\ldots,n \tag{8}$$

In order to determine a unique score for DMUs in Dynamic DEA framework, as [12] proposed the values of $\lambda_j (j=1,\ldots,n)$ must be equally selected ,i.e., $\lambda_1 = \lambda_2 = \ldots = \lambda_n$. Since we assume that the sum of efficiency scores are unity ($\sum_{j=1}^{n} \theta_j = 1$). Therefore, $\theta_j (j=1,\ldots,n)$ are determined by solving the following linear Equation system:

$$\begin{cases} \theta_j = \theta_j^{\min} \lambda + (1-\lambda)\theta_j^{\max} & j = 1,\ldots,n \\ \sum_{j=1}^{n} \theta_j = 1 \end{cases} \tag{9}$$

The value of λ can be easily obtained as follows:

$$1 = \sum_{j=1}^{n} \theta_j = \sum_{j=1}^{n} (\lambda \theta_j^{\min} + (1-\lambda)\theta_j^{\max}) \Rightarrow \lambda = \frac{1 - \sum_{j=1}^{n} \theta_j^{\max}}{\sum_{j=1}^{n} (\theta_j^{\min} - \theta_j^{\max})}. \tag{10}$$

Through substitution of (10) in (8), the values $\theta_j (j=1,\ldots,n)$ of can be determined. Now, with respect of the efficiency score θ_j, all DMUs can be fully ranked. In other words, a unit with greater efficiency score has a better rank.

NUMERICAL EXAMPLE

In order to highlight the applicability of the proposed ranking method, a numerical example consisting eleven DMUs in two periods are examined. The data set are derived from (Amirteimori, 2006) and consist of eleven gas companies located in eleven regions in Iran. Also, two six-month periods during 2003 and 2004 are illustrated. The data set are shown in Table 1, Table 2, Table 3 and Table 4. Variable input (x_{tj}) is budget and output (y_{tj}) is amount of piping. Another type of output which is the next period's input is revenue. In the beginning of each period, gas companies use the revenue of gas sold in the previous period as input in the current period. Applying model (6), the maximum and minimum efficiency scores $\theta_j^{\min}$ and $\theta_j^{\max}$ for all DMUs in whole periods are determined. Also, the efficiency score θ_j of each unit is exhibited. Finally, the units are ranked according to their integrated score. Table 5 shows these results.

Table 1: The normalized input data used in the application in period *t=1*

Companies	Budget ($x_j^{(t)}$)	Revenue of gas sold in previous period ($k_j^{(t-1)}$)
1	0.9625	0.9992
2	0.9265	0.9969
3	1	1
4	0.6009	0.8902
5	0.6617	0.6873
6	0.5464	0.4119
7	0.7287	0.5972
8	0.4038	0.1789
9	0.6186	0.3959
10	0.7309	0.3239
11	0.8250	0.9957

Table 2: The normalized output data used in the application in period *t=1*

Companies	Budget ($y_j^{(t)}$)	Revenue of gas sold in previous period ($k_j^{(t)}$)
1	1	0.9398
2	0.569	1
3	0.357	0.9907
4	0.5915	0.8996
5	0.937	0.5277
6	0.2558	0.4064
7	0.5177	0.7782
8	0.487	0.9415
9	0.3662	0.6134
10	0.8213	0.7324
11	0.1235	0.5191

Table 3: The normalized input data used in the application in period *t=2*

Companies	Budget ($x_j^{(t)}$)	Revenue of gas sold in previous period ($k_j^{(t-1)}$)
1	0.8973	0.9398
2	0.3884	1
3	0.7864	0.9907
4	0.6879	0.8996
5	1	0.5277
6	0.9662	0.4064
7	0.8261	0.7782
8	0.9169	0.9415
9	0.6223	0.6134
10	0.8813	0.7324
11	0.8876	0.5191

Table 4: The normalized output data used in the application in period *t=2*

Companies	Budget ($y_j^{(t)}$)	Revenue of gas sold in previous period ($k_j^{(t)}$)
1	1	0.1878
2	0.5325	0.8419
3	0.2555	1
4	0.9130	0.3372
5	0.9385	0.5516
6	0.2656	0.3555
7	0.5658	0.1811
8	0.4614	0.9852
9	0.3408	0.5262
10	0.8819	0.4786
11	0.7954	0.7394

Table 5: An equitable ranking of DMUs

Companies	Budget ($y_j^{(t)}$)	Revenue of gas sold in previous period ($k_j^{(t)}$)
1	0.0222	0.1287
2	0.0989	0.1747
3	0.0405	0.1615
4	0.0577	0.1407
5	0.0751	0.1370
6	0.0407	0.0739
7	0.0265	0.0839
8	0.0874	0.2454
9	0.0672	0.1315
10	0.0705	0.1283
11	0.0633	0.1238

Companies	θ_j	Rank
1	0.0649	9
2	0.1293	2
3	0.089	7
4	0.0898	6
5	0.0999	3
6	0.054	10
7	0.0495	11
8	0.1492	1
9	0.0930	5
10	0.0937	4
11	0.0875	8

CONCLUSION

This study proposed an alternative way for ranking DMUs in dynamic DEA (DDEA) framework. The proposed ranking method have different aspects of advantages. First of all, the method based on both pessimistic and optimistic attitude of dynamic DEA. Secondly, it can be more equitable than other methods based on only one attitude. Finally, a full ranking of all DMUs in the whole periods can be determined. Authors are hoped that this study make a small contribution to future development studied in dynamic DEA (DDEA).

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