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Obtaining a Unique Solution for the Cross Efficiency by Using the Lexicographic method

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Abstract

Cross efficiency is a method with the idea of peer evaluation instead of self-evaluation, and is used for evaluation and ranking Decision Making Units (DMUs) in Data Envelopment Analysis (DEA). Unlike most existing DEA ranking models which can only rank a subset of DMUs, for example non-efficient or extreme efficient DMUs, cross efficiency can rank all DMUs, even non-extreme ones. However, since DEA weights are generally not unique, cross-efficiency which uses optimal weights corresponding to evaluation of DMUs may not be unique either. This deficiency renders the cross efficiency method useless. However, the secondary goals proposed to deal with this deficiency of cross efficiency have such drawbacks themselves as well. In this paper we present a new secondary goal for cross efficiency method based on the lexicographic method. The main advantage of the proposed method is that with the possibility of existence of alternative optimal weights at the end of the secondary goal problem, the performance and the rank of DMUs will be constant, while the previous secondary goal methods don't offer any suggestions to deal with their alternative optimal weights.

Keywords*:* Cross Efficiency; Data Envelopment Analysis; Lexicographic Method; Ranking.

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1. Introduction

Data envelopment analysis (DEA) was originated in 1978 by Charnes et al. [1] and the first DEA model was called CCR (Charnes, Cooper and Rhodes) model. One of the main objectives of DEA is to measure the efficiency of a DMU (Decision Making Unit, e.g. firms, public agencies, universities and banks). It identifies a subset of efficient 'best practice' DMUs and for the remaining DMUs, the magnitude of their inefficiency is derived by comparison to a frontier constructed from the 'best practices'. Efficient DMUs are identified by an efficiency score equal to 1, and inefficient DMUs have efficiency scores less than 1. So, one of the main problems in DEA is ranking DMUs. Several authors have proposed methods for ranking the best performers ([2–9] among others). For a review of ranking methods, see [10]. Adler et.al [10] divided the ranking methods into some areas. One of them involves cross-efficiency method. Sexton et al. [11] first introduce the concept of a cross-efficiency measure in DEA. Cross-efficiency's main idea is to use DEA in a peer evaluation instead of only a self-evaluation. Self evaluation means that DMU is allowed to choose the most favorable prices or weights in order to maximize its efficiency. Peer evaluation means that DMU be evaluated by other DMUs' prices or weights. So cross efficiency solves the unreality in weights by noticing peer evaluation instead of using weight restrictions, which is usually used in DEA for obtaining acceptable prices or weights. This advantage and its power in discriminating all units is the reason for the wide use of cross efficiency for ranking the performance of DMUs. For example see [12-14]. One of the other main advantages of the cross efficiency method which must not to be ignored is its ability in ranking non-extreme DMUs. Most of the existing methods can only rank extreme DMUs. You now that almost all efficient frontier points are non-extreme. So for example if one may want to rank non-extreme target points corresponding to an inefficient point, which are attained from various target setting methods, the existence of a ranking method with ability of ranking non-extreme points is essential. However, there are still several limitations for utilizing the cross efficiency measure for evaluation. A problem that often arises, especially in the case of the extreme efficient units, is that we may have different optimal weights associated with the efficiency score of a given DMU. Overlooking the existence possibility of other optimal weights may lead to different ranks in cross efficiency, because an optimal weight can be favorable for a DMU and not favorable for the other one, and vice versa. So depending on which of the alternate optimal solutions is used, it may be possible to improve a DMU's rank, but generally only at the expense of worsening the others. Appa and Williams [15] show how such an event can occur by an illustration example. For this reason Sexton et al. [11] and Doyle and Green [16] propose the use of secondary goals to deal with the non-uniqueness issue. The secondary goal can be either aggressive or benevolent. In the case of the benevolent model, after maximizing the efficiency of the under evaluation DMU, the average efficiency of other DMUs would be maximized. But in the case of the aggressive model, the average efficiency of other DMUs would be minimized. Since efficiency of DMUs in the objective function was in fractional form neither Sexton et al. [11] nor Doyle and Green [16] method are able to find a weight set that achieves the minimum or the maximum of that non-linear fractional programming problem. Recently Liang et al. [17] extended the model of Doyle and Green (1994) by introducing three secondary objective functions which are linear or can be transformed into linear programming. They are minimizing total deviation from the ideal point, minimizing the maximum deviation from efficiency score and minimizing the mean absolute deviation. In fact they considered linear form of efficiency in their mind and tried to minimize the deviation of the efficiency score of DMUs to their ideals by using norm one and norm infinity in the first and second approach and also minimized the mean absolute deviation from those ideals. Albeit their proposed secondary objective functions do not have the problem of non-linearity, none of the since introduced secondary methods solve the problem of the alternative optimal solutions, and the worry about the variation of the performance of DMUs with respect to altering the optimal weights still exists. For this reason, in this paper we claim there is a need for selecting weights from among the optimal solutions of the optimal weights according to some criteria. In particular, we propose a procedure, based on the lexicographic method, which can eliminate the worry of existence of optimal weights. The lexicographic approach assumes a ranking of the objective functions according to their importance. Objective functions in our approach are the efficiency of DMUs whose importance is given by Decision Maker (DM). As you know, solving an MCDM problem with lexicographic method does not mean that there are not any alternative optimal solutions, but the optimal value of all optimal solutions is unique. So we can claim that despite the possibility of existence of alternative weights in our approach, the performance and therefore the rank of DMUs will be constant. This is the point about our method.

This paper has been organized as follows. In section 2, we briefly pay attention to the cross efficiency method and lexicographic method. Section 3 contains our proposal and in section 4, we have an illustrative example. Section 5 contains our conclusions.

2. Preliminarily

2.1. Cross Efficiency

Cross-efficiency method, when used for ranking and evaluating DMUs, involves two steps:

In the first step, the equivalent linear form of basic CCR model (1) is solved and optimal weights of inputs and outputs are obtained for each DMU (DMU t).

$$\theta_{tt} = Max\left\{ \frac{U^T Y_t}{V^T X_t} \middle| \frac{U^T Y_j}{V^T X_j} \le 1,\ j = 1,\ldots,n,\ V \ge \varepsilon,\ U \ge \varepsilon \right\} \tag{1}$$

Where X_j is the vector of inputs consumed by DMU_j, and Y_j is the vector of outputs produced by DMU_j.

In the second step, a cross-efficiency matrix like Table 1 is constructed.

Table 1: Cross Efficiency Matrix

	DMU 1	DMU 2	…	DMU n	Ave
DMU 1	θ_{11}	θ_{12}	…	θ_{1n}	s_1
DMU 2	θ_{21}	θ_{22}	…	θ_{2n}	s_2
:	:	:	:	:	
DMU n	θ_{n1}	θ_{n2}	…	θ_{nn}	s_n

Its arrays are the self-evaluated efficiencies determined by CCR model and cross efficiencies rated by its peers. The element in the diagonal (θ_{jj}) is the efficiency score for each DMU (DMU j) using CCR model. Other elements (θ_{ij}) are the cross efficiency of one DMU (DMU i) using the optimal weights of another (DMU j). So the cross-efficiency of DMU i, using the weights that DMU j has chosen in model (1), is then $\theta_{ij} = \frac{U^* Y_i}{V^* X_i},\ i, j = 1,2,\ldots,n$, where (*) denotes optimal values in model (1). The related score to each DMU can be obtained by averaging each row[†] of cross-efficiency matrix ($s_i = \frac{1}{n}\sum_{j=1}^{n} \theta_{ij},\ i = 1,2,\ldots,n$). The DMUs can be ranked according to their scores. The greater the score, the higher the rank .

But problem arises when the elements of cross-efficiency are altered by changing the optimal weight corresponding to an especial DMU. Changing the elements of cross-efficiency may change the average of rows and therefore the ranks. As mentioned before, for solving this problem, a secondary goal was considered. Sexton et al. [11] and Doyle and Green [16] proposed maximizing and minimizing the average of efficiency of the remaining DMUs in the case of benevolent and aggressive respectively.

[†] Notice that our cross efficiency matrix is transpose of the usual cross efficiency matrix which exists in the literature.

Since it lead to a fraction program problem, they suggested maximizing/minimizing the average unit, which is composed of remaining DMUs by averaging their inputs and outputs. In fact for obtaining the weights corresponding to DMU t in the benevolent case, they solved model 2:

$$s_t = Max\left\{ \frac{U^T\overline{Y}_t}{V^T\overline{X}_t} \middle| \frac{U^T\overline{Y}_t}{V^T\overline{X}_t} \le 1,\ \frac{U^TY_t}{V^TX_t} = \theta_{tt}, \frac{U^TY_j}{V^TX_j} \le 1, j=1,...,n,\ j \ne t, V \ge \varepsilon,\ U \ge \varepsilon \right\} \quad (2)$$

Where $(\overline{X}_t, \overline{Y}_t)$ denotes the average of DMUs excluding DMU t. For the aggressive case, Max would be converted to Min.

2.1. Lexicographic Method

Consider the following general form of a Multi Criteria Decision Making (MCDM) problem.

$$Max/\,Min\,\{(f_1(x),..., f_n(x)) \mid x \in X\} \qquad (3)$$

The approaches to generate the solution set of MCDM problems generally fall into two categories, scalarization methods and nonscalarizing methods. These approaches convert the MCDM into a single objective program, a sequence of single objective program, or other MCDM problems. Under some assumptions, solution sets of these new programs yield solutions of the original problem. Scalarization methods such as the weighted sum approach, distance-function-based approaches, the achievement function approach, the reference set approach and Goal programming, explicitly employ a scalarizing function to accomplish the conversion while nonscalarizing methods such as the max-ordering approach, the equitability approach and the lexicographic approach use other means. For more information about the MCDM approach see [18] and [19].

In lexicographic approach we consider the lexicographic order for objective function. The order may be based on the preferences of the DM. Let $f_1(x),..., f_n(x)$ be the objective function in this priority order. First in stage 1 maximize/minimize $f_1(x)$ over X. If the optimal solution for this problem is unique, accept it as the optimal solution of MCDM and terminate. Otherwise let K_1 denote the set of alternative optimal solution and go to stage 2. If the method does not terminate in stage r, we let K_r denote the set of alternative optimal solution of stage r problem. Then in stage r+1 maximize/minimize $f_{r+1}(x)$ over K_r. If the optimal solution for this problem is unique terminate. Otherwise go to stage r+2 and continue in the same way. The optimal solutions of the final stage are the lexicographic maximum/minimum optimal solution of (3).

3. Cross Efficiency and Lexicographic method

In this section we present a new secondary goal method, based on lexicographic method, for solving the problem of variation in the cross efficiency results which were mentioned before. Suppose that we evaluated DMU t by CCR model.
Logically we like to choose a weight from possible alternative optimal weights corresponding to DMU t such that the vector of efficiency of other DMUs is maximized. On the other hand we want to solve the following multi objective problem (if we have the benevolent viewpoint) corresponding to DMU t:

$$Max\left\{\frac{U^TY_j}{V^TX_j}, j=1,...,n, j\neq t \,\middle|\, \frac{U^TY_t}{V^TX_t}=\theta_{tt}, \frac{U^TY_j}{V^TX_j}\leq 1, j=1,...,n,\ j\neq t, V\geq\varepsilon,\ U\geq\varepsilon\right\} \quad (4)$$

There are various approaches for solving multi objective problem (4). Based upon the viewpoints of experts and decision makers, an especial approach can be selected. For example Sexton et al. [11] select the weighted sum approach. Then for prohibition of solving a nonlinear programming problem, they evaluated the average unit. One may use the goal programming method and try to minimize the deviation of each DMU’s efficiency to its goal by different norms. For example Hosseinzadeh Lotfi et al. [20] did a similar work for finding common set of weight in DEA. They minimized a weighted sum of deviation of DMU’s efficiencies to their goals which was 1. Their method leads to a nonlinear programming problem. Jahanshahloo et al. [6] did a similar work by using norm infinity. It leads to a nonlinear problem too. Cook and Zho [21] used norm 1 and norm infinity for deriving within-group common weights in DEA. They assumed that deviations worked as parameters and used Dinkelbach’s algorithm [22] and Consecutive interval search. Fuh-Hwa et al. [23] took into account a benchmark line and tried to minimize the sum of the total virtual gaps of only efficient DMUs for ranking them. If we use norm 1 for minimizing the deviation from efficiency scores of DMUs, a development version of their method as a secondary goal method for cross efficiency could be found. The obtained model can be considered as secondary goals similar to those proposed previously in the literature. But the problem still exists, “changing of the performances and ranks by altering the weights in the set of alternative optimal solution”. The reason for this may be the possibility of existence of tradeoffs between objectives. So we propose using the lexicographic method for (4). As stated before, for using lexicographic method we must have an order of functions. Since optimization of only one objective is considered in each step of lexicographic method, there are no tradeoffs between objective functions. In fact the priority ranking did not allow any existence of tradeoffs between objectives (which may be efficiencies or deviation from a goal). So it seems that using lexicographic method for finding the optimal set of (4) prohibits the problems mentioned before. Suppose that the perturbation (1,…,n) is considered. It means that more efficiency of DMU p has more importance than efficiency of DMU q for DM, where p is less than q. That means efficiency of DMU1 has the highest priority, and only in the case of multiple optimal solutions efficiency of DMU2 and the other efficiencies are considered. The algorithm of the proposed method is: (Without loss of generality, suppose t = n.).

Algorithm of the proposed method (respecting to DMU t):
Input: Optimal solution set K and objective functions f_j as follows:

$$K = \left\{ (U,V) \mid \frac{U^T Y_t}{V^T X_t} = \theta_{tt}, \frac{U^T Y_j}{V^T X_j} \le 1,\ j = 1,\dots,n-1,\ V \ge \varepsilon,\ U \ge \varepsilon \right\},$$

$$f_j(U,V) = \frac{U^T Y_j}{V^T X_j},\ j = 1,\dots,n-1$$

Initialization: Define $K_1 := K$ and $p := 1$.
Solve the single objective optimization problem

$$Max\left\{ f_p(U,V) \mid (U,V) \in K_p \right\}. \qquad (5)$$

While $p \le n-1$ do:

If (9) has a unique optimal solution (U_p^*, V_p^*), STOP, (U_p^*, V_p^*) is the unique optimal solution of the lexicographic optimization problem.

If $p = n-1$, STOP, the set of optimal solutions of the lexicographic optimization problem is:

$$\left\{ (U,V) \in K_{n-1} \mid f_{n-1}(U,V) = \max_{(U,V)\in K_{n-1}} f_{n-1}(U,V) \right\}.$$

Let $K_{p+1} := \left\{ (U,V) \in K_p \mid f_p(U,V) = \max_{(U,V)\in K_p} f_p(U,V) \right\}$ and let $p := p+1$.

End while.

Output: Set of lexicographically optimal weights for the cross efficiency.

The correctness of the algorithm can be verified by the following theorem which we present without proof, because it arises from the properties of lexicographic method.

Theorem: If (U_p^*, V_p^*) is a unique optimal solution of (5) with $p < n-1$, or if (U_p^*, V_p^*) is an optimal solution of (5) with $p = n-1$ then it is a lexicographically optimal solution.

If we notice the algorithm and theorem, two advantages of the proposed method can be inferred. First, since instead of solving (5) directly, a linear form of it can be solved; the problem of non-linearity which exists in some secondary methods is solved. Second, if the unique optimal weight exists for the secondary problem, there is not any ambiguity. But if there are some alternative optimal solutions, the optimal objective value for all optimal weights is constant and would not alter from one optimal weight to another, because all the optimal solutions were obtained by optimizing all efficiency functions $f_j(U,V)=\dfrac{U^T Y_j}{V^T X_j}$, $j=1,...,n-1$.

The hierarchy process among efficiency functions allowed us to solve lexicographic optimization problems sequentially, maximizing one efficiency function $f_p(U,V)$ at a time and using optimal objective values of $f_s(U,V), s<k$ as constraints, as shown in Algorithm. Therefore the arrays of the t'th (=n'th) column (t assumed to be the last DMU for simplicity in the algorithm) of the cross efficiency matrix is attained *uniquely* by

$$\left(\theta_{1n},...,\theta_{(n-1)n},\theta_{nn}\right)^t=\left(f_1(U^*,V^*),...,f_{n-1}(U^*,V^*),\theta_{nn}\right)^t.$$

So we can be sure that by altering the optimal weights, the performance and the rank of DMUs is not changed.

A few points are worth mentioning with respect to the proposed method:

1. We propose the algorithm in the benevolent case. In the case of aggressive, it is sufficient to change maximizing to minimizing in the mentioned algorithm and consider the reverse order for importance of objectives. The two advantages that were mentioned before about benevolent case exist here. So the obtained results of the proposed method are more stable than the result of other methods.
2. One may criticize this, saying that the DM preferences may influence the result. In reply to such viewpoints it must be said that each of the approaches for obtaining optimal solution of multi objective problems is based on an especial theory, and none of them can give a complete answer to multi objective problems. Each of them may give a part of the solution set. They may have some advantages over other methods, but the preference of the decision maker and the dominant conditions over the problem, limit the selection of the various methods. Multi objective methods can be divided into three categories according to which stage of the optimization the DM expresses his/her preferences: before, during or after the optimization, and in the previous methods if the DM wanted to obtain a unique weight from optimal weight set, he/she had to select the most preferred ones through some arbitrary processes and use some restricting criteria. In that case DM preferences also influence the results. We believe that using lexicographic method for the cross efficiency problem is the best choice.
3. In the previous methods, if DM wanted to experiment the other optimal solutions of the secondary function for evaluating the possible variation in the performances and ranks, he/she had to use a repetitive scheme. Also existence of some ending conditions for such algorithms may cause some of the optimal solutions to be disregarded. But since we know that the result is constant for all optimal solutions, such a procedure is meaningless.

4. Taking into account the inherent advantages of the cross efficiency, by eliminating its weakness for confronting alternative weights, it can be a good method for ranking all DMUs, extreme efficient DMUs, non-extreme efficient DMUs and also non-efficient DMUs. Most of the existing methods can be used for ranking an especial set of DMUs. For example in the model "Measure of inefficiency dominance" of Bardhan et al. [24] ranking can be only done for inefficient DMUs, and all ranking methods in the references [2-5,7-9] only can rank extreme efficient DMUs. But cross efficiency method can be used for ranking all DMUs.

4. Illustrative Example

In this section we use the lexicographic cross efficiency method for ranking six nursing homes (DMUs). Sexton et al. [11] considered this example for ranking by cross efficiency. Jahanshahloo et al. [8], Adler et al. [10], and Liang et al. [17] ranked these six DMUs using some ranking models as well. DMUs are compared over four variables: staff hours per day (StHr) and supplies per day (Supp) as inputs, total Medicare plus Medicaid reimbursed patient days (MCPD) and total private patient days (PPPD) as outputs. The raw data are presented in Table 2. In table 3 the results of the CCR, and the secondary goals of sexton et al. (benevolent), Liang et al. (three methods) and our proposed method (benevolent) are presented. We supposed that the order of the preferences is (1,2,…,6).

Table 2: Raw data for numerical example

	Inputs		Outputs	
	StHr	Supp	MCPD	PPPD
A	150	0.2	14000	3500
B	400	0.7	14000	21000
C	320	1.2	42000	10500
D	520	2.0	28000	42000
E	350	1.2	19000	25000
F	320	0.7	14000	15000

Table 3: CCR and Cross efficiency scores

	CCR Efficiency	Sexton et al	Liang et al. (1)	Liang et al. (2)	Liang et al. (3)	Proposed Method
A	1	1	1	1	1	1
B	1	1	0.9547	0.9617	0.9547	0.977341
C	1	0.974	0.8864	0.8759	0.8864	0.857955
D	1	0.955	1	1	1	1
E	0.9775	0.886	0.9742	0.9748	0.9742	0.975849
F	0.8675	0.847	0.8465	0.8499	0.8465	0.856984

5. Conclusion

A well-known method to evaluate the performances and to improve the discrimination among decision-making units is cross-efficiency. Cross-efficiency's main idea is to use DEA in a peer evaluation instead of using only a self-evaluation. But the existence of alternative optimal solutions, particularly in evaluation extreme DMUs, may cause changing of the evaluation results. For this reason, using the secondary goals to deal with the non-uniqueness issue was proposed. In order to obtain a suitable weight for an especial DMU, the efficiency vector of the remaining DMUs must be optimized over the alternative optimal solution of that DMU. For finding the optimal set of arises multi objective problem, various methods can be used, for example weighted sum, using various norms and so on. We introduced a new goal programming secondary goal. But neither that method nor the previous proposed methods in the literature could eliminate the drawbacks of the cross efficiency. So we present a new secondary goal for cross efficiency method based on the lexicographic method. Two advantages of the proposed method are its linearity and the fact that it does not change the performance and ranking result when we pass from one optimal weight to another. We used the proposed method for evaluating performance and ranking of some nursing homes.

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