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Canonical representation for approximating solution of fuzzy polynomial equations

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Abstract

In this paper, the concept of canonical representation is proposed to find fuzzy roots of fuzzy polynomial equations. We transform fuzzy polynomial equations to system of crisp polynomial equations, this transformation is perform by using canonical representation based on three parameters Value, Ambiguity and Fuzziness.

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1. Introduction

Polynomials play a major role in various areas such as mathematics, engineering and social sciences. The real solution of fuzzy polynomial equation and Dual fuzzy polynomial equation is investigated by Abbasbandy and Otadi [1-2].

In this paper, we are interested in finding fuzzy roots of fuzzy polynomial equation like $A_1x+A_2x^2+\ldots+A_nx^n=A_0$

Where A_0 , $A_1,\ldots A_n$ and x are fuzzy numbers .

In this paper, we propose a new method for solving fuzzy polynomial equation based on canonical representation which is introduced by Delgado et.al [4-5] they introduced three real indices called Value, Ambiguity and Fuzziness to obtain simple fuzzy numbers that could be used to represent more arbitrary fuzzy numbers. Therefore, we use these parameters and transform fuzzy polynomial by three crisp polynomial .

Then by solving this crisp system, we can find fuzzy roots of fuzzy polynomial.

2. Basic Concepts

Definition 2.1: A fuzzy number is a fuzzy set such as

$U : \mathrm{R} \rightarrow I = [0,1]$ that satisfies :

1- u is upper semi-continuous ,

2- $u(x)$ =0 outside some interval $[a,d]$;

3- There are real numbers b, c such that $a \leq b \leq c \leq d$ and

(a) $u(x)$ is monotonically increasing on $[a,b]$,

(b) $u(x)$ is monotonically decreasing on $[c,d]$,

(c) $u(x) =1$, $b \leq x \leq c$

The membership function u can be expressed as:

$$u(x)=\begin{cases} u_L(X) & a\le x\le b \\ 1 & b\le x\le c \\ u_R(x) & c\le x\le d \\ 0 & otherwise \end{cases}$$

where $u_L:[a,b]\to[0,1]$ and $u_R:[c,d]\to[0,1]$ are left and right membership functions of a fuzzy number u.

An equivalent parametric form is also given in [6] as follows.

Definition 2.2: A fuzzy number u in parametric form is a pair $(\underline{u},\overline{u})$ of functions $\underline{u}(r)$, $\overline{u}(r)$, $0\le r\le 1$,

That satisfies the following requirement:

1- $\underline{u}(r)$ is a bounded monotonically increasing left continuous function.

2 - $\overline{u}(r)$ is a bounded monotonically decreasing left continuous function,

3- $\underline{u}(r)\le\overline{u}(r)$, $0\le r\le 1$.

Definition 2.3.[4]: The increasing function $s:[0,1]\to[0,1]$ with property that $s(0)=0$ and $s(1)=1$ is a reducing function and if $\int_0^1 s(r)\,dr=\frac{1}{2}$, we call it a regular reducing function.

Definition 2.4: [4] Let u be a fuzzy number with parametric form $(\underline{u}(r),\overline{u}(r))$ and let $s:[0,1]\to[0,1]$ be a reducing function. Then the Value of u (with respect to s) is

$$Va\,l(a)=\int_0^1 s(r)[\underline{u}(r)+\overline{u}(r)]\,dr,$$

Then the Ambiguity of u (with respect to s) is

$$Amb(u) = \int_0^1 s(r)[\,\overline{u}(r) - \underline{u}(r)]\,dr$$

and for *s(r) = r*, the Fuzziness can be assessed by

$$Fuzz(u) = \int_0^{\frac{1}{2}} [\,\overline{u}(r) - \underline{u}(r)]\,dr + \int_{\frac{1}{2}}^1 [\,\underline{u}(r) - \overline{u}(r)]\,dr.$$

3. Approximate Solution Of Fuzzy Polynomial Equation.

In this section we are going to split a fuzzy polynomial and find its approximate solution by solving the associated split system.

Definition 3.1: We define associated split system as follow:

(1) $Val(A_1x+A_2x^2+\ldots A_nx^n)=Val(A_0)$

(2) $Amb(A_1x+A_2x^2+\ldots A_nx^n)=Amb(A_0)$

(3) $Fuzz(A_1x+A_2x^2+\ldots A_nx^n)=Fuzz(A_0)$

Then we have:

(4) $Val(A_1)\,Val(x) + Val(A_2)\,Val(x^2) +\ldots+ Val(A_n)\,Val(x^n)=Val(A_0)$

(5) $Amb(A_1)Amb(x)+Amb(A_2)Amb(x^2)+\ldots+ Amb(A_n)Amb(x^n)=Amb(A_0)$

(6) $Fuzz(A_1)Fuzz(x)+Fuzz(A_2)Fuzz(x^2)+\ldots+Fuzz(A_n)Fuzz(x^n)=Fuzz(A_0)$

By solving (4),(5),(6) we have (Val(x),Amb(x),Fuzz(x));

Then we want to obtain a symmetric trapezoidal representation for *x*. let $T=(b_1,b_2,b_3,b_4)$ be a symmetrical trapezoidal number and suppose that its defining parameters are written as $b_1=m-c-d$, $b_2=m-c$, $b_3=m+c$, $b_4=m+c+d$, and s(r) =i(r)=r again. It is easy to see that Val(T)=m, Amb(T)= $\frac{d}{3}$ +c, and Fuzz(T)= $\frac{d}{2}$, solving for c and d, we obtain

$$d = 2\,Fuzz(T) \qquad , \qquad c=Amb(T) - \left(\frac{2}{3}\right)Fuzz(T).$$

Suppose now we are given a fuzzy number x with parameters Val(x)=x_v , Amb(x)=x_a , and Fuzz(x)=x_f , then from the above results we can construct a symmetrical trapezoidal number T such that Val(T)=x_v , Amb(T)=x_a , and Fuzz(T)=x_f provided $c \geq 0$, or equivalently, $x_a - \frac{2}{3} x_f \geq 0$.

Otherwise, $x_a - \frac{2}{3} x_f < 0$, x will not have a canonical trapezoidal representation. To define a canonical representation for fuzzy numbers for which c < 0, we will make use of what we call (symmetrical) quasi-trapezoidal numbers. The parametric representation of a typical quasi-trapezoidal number is as follow;

$(\underline{u}(r), \overline{u}(r))$:

$$\underline{u}(r)=\begin{cases} m-s-t & if \quad r=0 \\ \frac{rt}{h}-t-s & if \quad 0<r<h \\ m-s & if \quad h \leq r \leq 1 \end{cases} \qquad \overline{u}(r)=\begin{cases} m+s+t & if \quad r=0 \\ \frac{-rt}{h}+t+s & if \quad 0<r<h \\ m+s & if \quad h \leq r \leq 1 \end{cases}$$

Carrying out the appropriate integrations, we easily obtain the parameters for w, Val(w)=m, Amb(w)=$\frac{th^3}{3}$ +s, [3]

$$\text{Fuzz}(w)=\begin{cases} th & if \quad h \leq \frac{1}{2} \\ t\left(\frac{-1}{2t}+2-h\right) & if \quad h \geq \frac{1}{2} \end{cases}$$

Observe that if $h \leq \frac{1}{2}$, then for s near 0, the ratio $\frac{Fuzz\ (w)}{Amb\ (w)}$ approaches $\frac{3}{h^2}$, and, consequently it will the case that whenever $\frac{Fuzz\ (w)}{Amb\ (w)}$ is large, we can find non-negative number h,s,t to construct a quasi-trapezoidal number w with the same Value, Ambiguity, and Fuzziness as the original fuzzy number. The challenge here, however, is that different combinations of h,s,t can yield the same value of Val(w), Amb(w), and Fuzz(w). if a particular canonical formula is required, then one might consider that for which h is maximum,i.e, the "most trapezoidal" quasi-trapezoidal feasible number. In the case, we have

Val(w)= th, Amb(w) = $\frac{th^3}{3}$ and Fuzz (w) = $\frac{h^2}{3} + s$.

4. Numerical Examples

Example 4.1: Consider the following fuzzy polynomial

(0, 1, 2,) x^2+ (2, 3, 5) x= (2 ,7 ,13)

where x is a fuzzy number.

Now according to section 3 obtain:

$x_v=2$, $x_a=\frac{1}{3}$ and $x_f=\frac{1}{2}$.

It is easy to see that m = 2, c = 0, d = 1

we obtain x=(1,2,2,3).

Example 4. 2. [7]: Suppose a corporation wishes to set aside around one million dollars ((A=1, 0.2, 0.2)) to be invested at interest rate R so that after one year they may withdraw approximately 250,000 dollars

(S_1=(0.25, 0.05, 0.05)) and after 2 years, the amount that is left will accumulate to about 900,000 dollars (S_2=(0.9, 0.3, 0.3)). Find R so that A will be sufficient to cover about S_1 and S_2 . R will be a fuzzy number whose support lies in [0,1].

After one year the amount in the account will be A+AR .

After withdrawing S_1 the amount at start of the second year is

$(A-S_1)+AR$.

At the end of the second year the accumulated total is

$[(A-S_1)+ AR]+ [(A-S_1)+AR]R$ or AR^2+BR+D

where B=2A-S and D=A-S

since multiplication distributes over addition for positive fuzzy numbers.

Therefor, we must solve

$AR^2+BR+D=S_2$

or

$(1, 0.2, 0.2)R^2+(1.75, 0.45, 0.45)R+(0.75, 0.25, 0.25)=(0.9, 0.3, 0.3)$

where R is fuzzy number.

Now according to section 3 obtain:

$X_v= 0.08$, $x_a=3.66\times10^{-4}$, $x_f = 0.002$

it is easy to see that m=0.08 , c=0.001 , d=0.004 we obtain:

x=(0.075, 0.079, 0.081, 0.085).

5. Conclusion

Solving fully fuzzy polynomial equations (FFPEs) by using Canonical representation(CR) is presented in this paper.We transformed fuzzy polynomial equations to system of crisp polynomial equations, this transformation is perform by using canonical representation based on three parameters Value, Ambiguity and Fuzziness.

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