



A method for solving fully fuzzy linear system with trapezoidal fuzzy numbers

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Abstract

Different methods have been proposed for finding the non-negative solution of fully fuzzy linear system (FFLS) i.e. fuzzy linear system with fuzzy coefficients involving fuzzy variables. To the best of our knowledge, there is no method in the literature for finding the non-negative solution of a FFLS without any restriction on the coefficient matrix. In this paper a new computational method is proposed to solve FFLS without any restriction on the coefficient matrix by representing all the parameters as trapezoidal fuzzy numbers.

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1. Introduction

One field of applied mathematics that has many applications in various areas of science is solving a system of linear equations. Systems of simultaneous linear equations play a major role in various areas such as operational research, physics, statistics, engineering and social sciences. When the estimation of the system coefficients is imprecise and only some vague knowledge about the actual values of the parameters is available, it may be convenient to represent some or all of them with fuzzy numbers [22]. Fuzzy number arithmetic is widely applied and useful in computation of linear system whose parameters are all or partially represented by fuzzy numbers.

Dubois and Prade [11,12] investigated two definitions of a system of fuzzy linear equations, consisting of system of tolerance constraints and system of approximate equalities. The simplest method for finding a solution for this system is creating scenarios for the fuzzy system, which is a realization of fuzzy systems. Based on these actual scenarios, Buckley and Qu [7] extended several methods for this category and proved their equivalence. But their approaches are not practicable, because infinite number of scenarios can be driven for a fully fuzzy linear system (FFLS).

Zhao and Govind [23] studied the algebraic equations involving generalized fuzzy numbers (which includes fuzzy numbers, fuzzy intervals, crisp numbers and interval numbers) with continuous membership functions. A general model for solving a fuzzy linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy vector was first proposed by Friedman et al. [13]. Friedman et al. [14] investigated a dual fuzzy linear system by mean of nonnegative matrix theory.

Allahviranloo [4] proposed solution of a fuzzy linear system by using iterative method (Jacobi and Gauss Seidel methods), later on the same author proposed

the solution of such system using Successive over relaxation iterative method [5] and Adomian decomposition method [6]. Abbasbandy et al. [3] proposed the Conjugate gradient method, for solving fuzzy symmetric positive definite system of linear equation. Dehghan and Hashemi [9] extended the Adomian decomposition method [6], to find the positive fuzzy vector solution of fully fuzzy linear system. Dehghan et al. [8] proposed classic methods such as Cramer's rule, Gaussian elimination method, LU decomposition method from linear algebra and linear programming for finding the approximated solution of a fully fuzzy linear systems.

Abbasbandy and Jafarian [2] applied Steepest descent method for approximation of the unique solution of fuzzy system of linear equation. Abbasbandy et al. [1] used LU decomposition method for solving fuzzy system of linear equation when the coefficient matrix is symmetric positive definite.

Muzzioli and Reynaerts [17] pointed out that although several investigations are reported in the literature of the solution of fuzzy systems, very few methods are available for the practical solution of a fuzzy linear system. They introduced an algorithm to find vector solution by transforming the system $A_1x + B_1 = A_2x + B_2$ into the FFLS $Ax = B$ where $A = A_1 - A_2$ and $B = B_1 - B_2$. Dehghan and Hashemi [10] modified the existing methods employed by Allahviranloo [4] for solving fuzzy linear systems. Mosleh et al. [16] proposed a method to find the solution of fully fuzzy linear system of the form $Ax + B = Cx + D$ with A, C square matrices of fuzzy coefficients and B, D fuzzy number vectors and the unknown vector x is vector consisting of n fuzzy numbers. Nasserri et al. [18] used a certain decomposition methods of the coefficient matrix for solving fully fuzzy linear system of equations. Yin and Wang [21] considered the general case of Splitting iterative methods for solving fuzzy system of linear equation.

Sun and Guo [20] proposed a general model for solving fuzzy linear systems and general dual fuzzy linear systems. Nasseri and Zahmatkesh [19] proposed a new method for computing the non-negative solution of fully fuzzy linear system of equations.

In this paper, a new computational method is proposed to find the non negative solution of FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$, where \tilde{A} is an arbitrary fuzzy matrix, \tilde{x} and \tilde{b} are fuzzy vectors with appropriate sizes, is proposed without any restriction on the coefficient matrix by representing all the parameters as trapezoidal fuzzy numbers as trapezoidal fuzzy numbers span entirely all the triangular fuzzy numbers, thus there are more generic.

The rest of this paper is organized as follows: In section 2, shortcomings of the existing methods to solve FFLS are described. In section 3 some basic definitions are reviewed. In Section 4 a new method is proposed for solving FFLS. In section 5 numerical examples are solved to show the efficiency of the proposed method. Section 6 ends this paper with a conclusion.

2. Shortcomings of existing methods

In this section the shortcomings in the existing methods [1-10,18,19] are pointed out:

1. The existing methods presume the non negativity of the coefficient matrix. This restriction creates difficulty in using the existing methods to solve a FFLS occurring in real life situations.
2. In all the existing methods, it is assumed that the system of equations is consistent and then the methods are developed i.e. consistency of the FFLS cannot be checked using the existing methods.
3. Using the existing methods it is not possible to check that the obtained solution is unique or not.

To overcome the above shortcomings, in section 4, a new computational method is proposed for solving a FFLS.

3. Preliminaries

In this section some basic definitions of fuzzy set theory are reviewed [15]

Definition 3.1. The Characteristic function $\mu_{\tilde{A}}$ of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X . This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \rightarrow [0,1]$. The assigned value indicate the membership grade of the element in the set A .

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ is called a fuzzy set.

Definition 3.2. A fuzzy set \tilde{A} , defined on the universal set of real number \mathbf{R} , is said to be a fuzzy number if its membership function has the following characteristics:

(i) \tilde{A} is convex i.e.,

$$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \forall x_1, x_2 \in \mathbf{R}, \forall \lambda \in [0,1]$$

(ii) \tilde{A} is normal i.e., $\exists x_0 \in \mathbf{R}$ such that $\mu_{\tilde{A}}(x_0) = 1$

(iii) $\mu_{\tilde{A}}$ is piecewise continuous.

Definition 3.3. A fuzzy number \tilde{A} is said to be non-negative fuzzy number if and only $\mu_{\tilde{A}}(x) = 0 \forall x < 0$.

Definition 3.4. A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < m - \alpha \\ 1 - \frac{m - x}{\alpha}, & m - \alpha \leq x \leq m, \alpha > 0 \\ 1 & m < x < n \\ 1 - \frac{x - n}{\beta}, & n \leq x \leq n + \beta, \beta > 0 \\ 0 & x > n + \beta \end{cases}$$

Definition 3.5. A trapezoidal fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be non-negative trapezoidal fuzzy number if and only if $m - \alpha \geq 0$.

Definition 3.6. A trapezoidal fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be zero trapezoidal fuzzy number if and only if $m = 0, n = 0$ and $\alpha, \beta = 0$

Definition 3.7. Two fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ and $\tilde{B} = (p, q, \gamma, \delta)$ are said to be equal i.e. $\tilde{A} = \tilde{B}$ if and only if $m = p, n = q, \alpha = \gamma$ and $\beta = \delta$.

Definition 3.8. A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix, if each element of this matrix is a fuzzy number. It will be positive (negative) and denoted by $\tilde{A} > 0$ ($\tilde{A} < 0$) if each element of this matrix be positive (negative). It will be non positive (non negative) and denoted by $\tilde{A} \leq 0$ ($\tilde{A} \geq 0$) if each element be non positive (non negative). We may represent $n \times m$ fuzzy matrix of trapezoidal fuzzy numbers by $\tilde{A} = (\tilde{a}_{ij})_{n \times m}$ where $\tilde{a}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$

Definition 3.9. Let $\tilde{A} = (\tilde{a}_{ij})$ and $\tilde{B} = (\tilde{b}_{ij})$ be two $m \times n$ and $n \times p$ fuzzy matrices. We define

$$\tilde{C} = \tilde{A} \otimes \tilde{B} = (\tilde{c}_{ij}) \text{ which is the } m \times p \text{ matrix where } \tilde{c}_{ij} = \sum_{k=1, \dots, n}^{\oplus} \tilde{a}_{ik} \otimes \tilde{b}_{kj}$$

3.2. Arithmetic operations on trapezoidal fuzzy numbers

In this subsection addition and multiplication operations between two trapezoidal fuzzy numbers are reviewed [15].

Let $\tilde{A}_1 = (m, n, \alpha, \beta)$ and $\tilde{A}_2 = (p, q, \gamma, \delta)$ be two trapezoidal fuzzy numbers then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (m, n, \alpha, \beta) \oplus (p, q, \gamma, \delta) = (m + n, p + q, \alpha + \gamma, \beta + \delta)$

(ii) $-\tilde{A}_1 = -(m, n, \alpha, \beta) = (-n, -m, \beta, \alpha)$

(iii) If $\tilde{A}_2 \geq 0$ then,

$$\tilde{A}_1 \otimes \tilde{A}_2 = (m, n, \alpha, \beta) \otimes (p, q, \gamma, \delta) = (\min(mp, mq), \max(np, nq), \theta, \pi)$$

$$\text{where } \theta = \min(mp, mq) - \min((m - \alpha)(p - \gamma), (m - \alpha)(q + \delta))$$

$$\pi = \max((n + \beta)(p - \gamma), (n + \beta)(q + \delta)) - \max(np, nq)$$

Remark 3.1. In this paper at all places minimum and maximum are represented by min and max respectively.

4. Proposed method

In this section a new computational method is proposed to find the solutions of a

FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where $\tilde{x} \geq 0$.

The steps of the proposed method are as follows:

Step1: Substituting $\tilde{A} = (\tilde{a}_{ij})_{n \times n}$, $\tilde{x} = (\tilde{x}_j)_{n \times 1}$ and $\tilde{b} = (\tilde{b}_j)_{n \times 1}$ the FFLS may be written as:

$$\sum_{j=1, \dots, n}^{\oplus} \tilde{a}_{ij} \otimes \tilde{x}_j = \tilde{b}_i \quad \forall i = 1, 2, \dots, n.$$

Step 2: If all the parameters \tilde{a}_{ij} , \tilde{x}_j and \tilde{b}_j are represented by trapezoidal fuzzy numbers

$$(m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}), \quad (x_j, y_j, z_j, w_j) \geq 0 \text{ and}$$

(b_i, g_i, h_i, k_i) respectively, then the FFLS obtained in step 1, may be written as:

$$\sum_{j=1, 2, \dots, n}^{\oplus} (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}) \otimes (x_j, y_j, z_j, w_j) = (b_i, g_i, h_i, k_i) \quad \forall i = 1, 2, \dots, n$$

Step 3: Assuming $(m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}) \otimes (x_j, y_j, z_j, w_j) = (f_{ij}, p_{ij}, q_{ij}, r_{ij})$, the

FFLS, obtained in step 2, may be written as:

$$\sum_{j=1, 2, \dots, n}^{\oplus} (f_{ij}, p_{ij}, q_{ij}, r_{ij}) = (b_i, g_i, h_i, k_i) \quad \forall i = 1, 2, \dots, n$$

where $(f_{ij}, p_{ij}, q_{ij}, r_{ij}) = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij}) \otimes (x_j, y_j, z_j, w_j)$ i.e.

$$f_{ij} = \min(m_{ij}x_j, m_{ij}y_j) = \begin{cases} m_{ij}x_j & \text{if } m_{ij} \geq 0 \\ m_{ij}y_j & \text{if } m_{ij} < 0 \end{cases}$$

$$p_{ij} = \max(n_{ij}x_j, n_{ij}y_j) = \begin{cases} n_{ij}y_j & \text{if } n_{ij} \geq 0 \\ n_{ij}x_j & \text{if } n_{ij} < 0 \end{cases}$$

$$q_{ij} = f_{ij} - \min((m_{ij} - \alpha_{ij})(x_j - z_j), (m_{ij} - \alpha_{ij})(y_j + w_j))$$

$$\min((m_{ij} - \alpha_{ij})(x_j - z_j), (m_{ij} - \alpha_{ij})(y_j + w_j)) = \begin{cases} (m_{ij} - \alpha_{ij})(x_j - z_j) & \text{if } m_{ij} - \alpha_{ij} \geq 0 \\ (m_{ij} - \alpha_{ij})(x_j + w_j) & \text{if } m_{ij} - \alpha_{ij} < 0 \end{cases}$$

$$r_{ij} = \max((n_{ij} + \beta_{ij})(x_j - z_j), (n_{ij} + \beta_{ij})(y_j + w_j)) - p_{ij}$$

$$\max((n_{ij} + \beta_{ij})(x_j - z_j), (n_{ij} + \beta_{ij})(y_j + w_j)) = \begin{cases} (n_{ij} + \beta_{ij})(x_j - z_j) & \text{if } n_{ij} + \beta_{ij} < 0 \\ (n_{ij} + \beta_{ij})(y_j + w_j) & \text{if } n_{ij} + \beta_{ij} \geq 0 \end{cases}$$

Step 4: Using the arithmetic operations, defined in section 3, the FFLS, obtained in step 3, may be written as :

$$\left(\sum_{j=1}^n f_{ij}, \sum_{j=1}^n p_{ij}, \sum_{j=1}^n q_{ij}, \sum_{j=1}^n r_{ij}\right) = (b_i, g_i, h_i, k_i) \quad \forall i = 1, 2, \dots, n$$

Step 5: The FFLS, obtained in step 4, may be converted into the following crisp linear system of equations of order $4n \times 4n$

$$\begin{cases} \sum_{j=1}^n f_{ij} = b_i & \forall i = 1, 2, \dots, n \\ \sum_{j=1}^n p_{ij} = g_i & \forall i = 1, 2, \dots, n \\ \sum_{j=1}^n q_{ij} = h_i & \forall i = 1, 2, \dots, n \\ \sum_{j=1}^n r_{ij} = k_i & \forall i = 1, 2, \dots, n \end{cases} \tag{1}$$

Step 6: Solve the crisp system of equation by using any classical method like LU decomposition or Matrix inversion or Cramer’s rule or Row reduced echelon

form or iterative methods like Gauss seidel, Jacobi, Adomian decomposition etc. as described in the literature. Check that the crisp linear system of equations obtained from the system of equation in step 5 is consistent or not.

Case (i): If the system of equations is inconsistent then the given FFLS is inconsistent.

Case (ii): If the system of equations is consistent then find solution of (1) i.e. evaluate $x_j, y_j, z_j, w_j \forall j = 1, 2, \dots, n$ and Go to step 7.

Step7: Check the Feasibility of the computed solution i.e.

$$x_j - z_j \geq 0, y_j \geq x_j \text{ and } z_j, w_j \geq 0 \quad \forall j = 1, 2, \dots, n.$$

Case (i): If the system of equations is not feasible then the given FFLS is inconsistent and has no feasible fuzzy solution.

Case (ii): If the system of equation is feasible then find the solution of FFLS as:

$$\tilde{x}_j = (x_j, y_j, z_j, w_j) \forall j = 1, 2, \dots, n.$$

Remark 4.1. The $n \times n$ FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$ will have a feasible non negative solution $\tilde{x}_j = (x_j, y_j, z_j, w_j) \forall j = 1, 2, \dots, n$ iff $\tilde{x}_j = (x_j, y_j, z_j, w_j) \geq 0$ i.e.

$$x_j - z_j \geq 0, y_j \geq x_j \text{ and } z_j, w_j \geq 0 \quad \forall j = 1, 2, \dots, n.$$

5. Numerical examples

Example 5.1. Let us consider the following FFLS and solve it by the proposed method

$$(3, 6, 2, 2) \otimes (x_1, y_1, z_1, w_1) \oplus (4, 6, 1, 2) \otimes (x_2, y_2, z_2, w_2) = (27, 66, 20, 70)$$

$$(1,2,6,2) \otimes (x_1, y_1, z_1, w_1) \oplus (4,5,2,2) \otimes (x_2, y_2, z_2, w_2) = (17,37,58,55)$$

$$(x_1, y_1, z_1, w_1) \geq 0, (x_2, y_2, z_2, w_2) \geq 0.$$

Solution: Using the proposed method, the given 2×2 FFLS is converted into a 8×8 linear system of equation. From equation (1) we get,

$$3x_1 + 4x_2 = 27$$

$$x_1 + 4x_2 = 17$$

$$6y_1 + 6y_2 = 66$$

$$2y_1 + 5y_2 = 37$$

$$27 - (x_1 - z_1) - 3(x_2 - z_2) = 20$$

$$17 + 5(y_1 + w_1) - 2(x_2 - z_2) = 58$$

$$8(y_1 + w_1) + 8(y_2 + w_2) - 66 = 70$$

$$4(y_1 + w_1) + 7(y_2 + w_2) - 37 = 55$$

The matrix form of this system of equations is:

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 5 & 0 & 0 \\ 1 & 0 & -1 & 0 & 3 & 0 & -3 & 0 \\ 0 & 5 & 0 & 5 & -2 & 0 & 2 & 0 \\ 0 & 8 & 0 & 8 & 0 & 8 & 0 & 8 \\ 0 & 4 & 0 & 4 & 0 & 7 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \\ x_2 \\ y_2 \\ z_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} 27 \\ 17 \\ 66 \\ 37 \\ 7 \\ 41 \\ 136 \\ 92 \end{bmatrix}$$

On solving the above linear system of equations directly by matrix inversion we obtain the following unique solution:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \\ x_2 \\ y_2 \\ z_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 4 \\ 3 \\ 3 \\ 5 \\ 1 \\ 3 \end{bmatrix}$$

Since $x_j - z_j \geq 0, y_j \geq x_j$ and $z_j, w_j \geq 0 \quad \forall j = 1, 2$. The given FFLS is feasible. We compute the solution as follows: $\tilde{x}_1 = (x_1, y_1, z_1, w_1) = (5, 6, 4, 3)$ and $\tilde{x}_2 = (x_2, y_2, z_2, w_2) = (3, 5, 1, 3)$

Example 5.2. Let us consider the following FFLS and solve it by the proposed method

$$(-1, 2, 2, 2) \otimes (x_1, y_1, z_1, w_1) \oplus (3, 4, 1, 1) \otimes (x_2, y_2, z_2, w_2) = (13, 42, 25, 42)$$

$$(7, 10, 3, 2) \otimes (x_1, y_1, z_1, w_1) \oplus (0, 1, 5, 2) \otimes (x_2, y_2, z_2, w_2) = (21, 58, 77, 50)$$

$$(x_1, y_1, z_1, w_1) \geq 0, (x_2, y_2, z_2, w_2) \geq 0.$$

Solution: Using the proposed method, the given 2×2 FFLS is converted into a 8×8 linear system of equation. From equation (1) we get,

$$-y_1 + 3x_2 = 13$$

$$7x_1 + 0x_2 = 21$$

$$2y_1 + 4y_2 = 42$$

$$10y_1 + y_2 = 58$$

$$13 + 3(y_1 + w_1) - 2(x_2 - z_2) = 25$$

$$21 - 4(x_1 - z_1) + 5(y_2 + w_2) = 77$$

$$4(y_1 + w_1) + 5(y_2 + w_2) - 42 = 42$$

$$12(y_1 + w_1) + 3(y_2 + w_2) - 58 = 50$$

The matrix form of this system of equations is:

$$\begin{bmatrix} 0 & -1 & 0 & 0 & 3 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 3 & -2 & 0 & 2 & 0 \\ -4 & 0 & 4 & 0 & 0 & 5 & 0 & 5 \\ 0 & 4 & 0 & 4 & 0 & 5 & 0 & 5 \\ 0 & 12 & 0 & 12 & 0 & 3 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \\ x_2 \\ y_2 \\ z_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} 13 \\ 21 \\ 42 \\ 58 \\ 12 \\ 56 \\ 84 \\ 108 \end{bmatrix}$$

On solving the above linear system of equations directly by matrix inversion we obtain the following unique solution:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \\ x_2 \\ y_2 \\ z_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 2 \\ 1 \\ 6 \\ 8 \\ 3 \\ 4 \end{bmatrix}$$

Since $x_j - z_j \geq 0$, $y_j \geq x_j$ and $z_j, w_j \geq 0 \quad \forall j = 1, 2$. The given FFLS is feasible. We compute the solution as follows: $\tilde{x}_1 = (x_1, y_1, z_1, w_1) = (3, 5, 2, 1)$ and $\tilde{x}_2 = (x_2, y_2, z_2, w_2) = (6, 8, 3, 4)$

6. Conclusion

In this paper, a new computational method for finding the non negative solutions of FFLS having trapezoidal fuzzy numbers and an arbitrary coefficient matrix, is proposed. The proposed method is easy to understand and apply in real life situations. The method is illustrated with the help of numerical examples. The method realizes the objective of the paper to solve a FFLS with no restrictions on the coefficients.

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